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Effects of eccentric circular perforation on thermal vibration of circular graphene sheets using translational addition theorem

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ABSTRACT

In this article, nonlocal thin plate theory of Eringen is employed to investigate effects of thermal environment on behavior of freely vibrating circular single layer graphene sheet containing a circular perforation of arbitrary size and location. In order to analytically solve equation of motion, the separation of variables method in conjunction with the translational addition theorem for Bessel functions is used. Accuracy and stability of results are verified by the literature. The behavior of sheets in low and high temperature conditions is considered. The influences of temperature change and various geometrical parameters on the natural frequencies are investigated by considering size-dependent material properties. In some cases, thermal buckling phenomenon was observed. Results show that the temperature changes play an important role in free vibration behavior of eccentric circular graphene sheets.

1. Introduction

Outstanding properties of single layer graphene sheets (SLGSs) as compared to the conventional structural materials lead to many potential applications for structures which are composed of these materials. Due to having very high natural frequencies up to gigahertz range [2], the SLGSs have gained much popularity and strong applications in nanomechanical resonators as one of the most important nano-electro- mechanical systems (NEMS).

Also, because SLGSs are very sensitive against thermal loads or environments, they can be used as a thermal nano-switchs in the electrical circuits that need high accuracy to switching the current in special set points. Also, recently, biomechanical researchers find that the annular nano-filters can be used as a trap for cancer cells to destroy them. Besides, the exceptional thermal properties of graphene have been used to improve the thermal stability and conductivity in nanostructures. The 2D geometry of graphene-base materials may offer lower interfacial thermal resistance and thus provide higher thermal conductivity in the nanostructures [3]. Therefore, understanding effects of temperature changes on natural frequencies of single-layered graphene sheets is an important problem.

During the production process of a nanostructure including graphene sheet, constrain conditions such as pin hole may be created in a SLGS. Also, perforations or cutouts of various shapes may be created in the SLGSs for various practical purposes such as vibration control, structural parameter identification and damage recognition. Some of these constraints and cutouts on a circular graphene sheet can be considered as an eccentric circular hole with arbitrary size and location. Hence, analyzing behavior of circular graphene sheet with an eccentric hole is important.

Many investigations have been carried out on free vibration analysis of graphene sheets. Aghababaei and Reddy [4] reformulated the third-order shear deformation plate theory of Reddy by using the nonlocal linear elasticity theory of Eringen that has ability to capture the both small scale effects and quadratic variation of shear strain and consequently shear stress through the plate thickness. Murmu and Pradhan [5], implemented nonlocal elasticity theory to study the vibration response of single-layered graphene sheets. They investigated influence of the surrounding elastic medium on the fundamental frequencies of the SLGS. Their numerical results show that the fundamental frequencies of SLGS are strongly dependent on the small scale coefficients. Pradhan and Phadikar [6] carried out vibration analysis of multilayered graphene sheets embedded in polymer matrix employing nonlocal continuum mechanics. They showed that nonlocal effect is quite significant and needs to be included in the continuum model of graphene sheet.

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Some researchers have studied on free vibration analysis of graphene sheets in thermal environments or loads. Mohammadi et al. [7] studied free vibration of annular and circular graphene sheets under radial compression load and temperature change. The graphene sheet rests on an elastic foundation and the nonlocal elasticity theory is used to model the problem. Prasanna Kumar et al. [8] presented the thermal vibration analysis of single-layer graphene sheet embedded in polymer elastic medium, using the plate theory and nonlocal continuum mechanics for small scale effects. Nonlinear vibration behavior is presented for a simply supported, rectangular, single layer graphene sheet in thermal environments by Shen et al. [9]. The thermal effects are also included and the material properties are assumed to be temperature-dependent and are obtained from molecular dynamics simulations. Wang and Hu [10,11] investigated the thermal vibration of rectangular and circular single-layered graphene sheets using a rectangular nonlocal elastic plate model with quantum effects. They found that the quantum effect is more important for the thermal vibration of higher modes in the case of smaller sides and lower temperature. Mandal and Pradhan [12] analyzed the effect of magneto-thermal environment on the transverse vibration of magnetically sensitive single-layered graphene sheets based on nonlocal plate theory. The equations of motion are derived considering the Lorentz magnetic force obtained from Maxwell's relationship and thermal elasticity. The governing differential equations are solved employing a differential quadrature method. Fazelzadeh and Pouresmaeeli [13] analyzed thermo-mechanical vibration of double-orthotropic nanoplates embedded in elastic medium. Asemi and Farajpour [14] analyzed axisymmetric vibration properties of circular single-layered graphene sheet embedded in a polymer matrix under thermo-mechanical loading. They considered both surface and nonlocal effects. Gurtin–Murdoch continuum elasticity in conjunction with the nonlocal elasticity theory is used to develop a modified continuum plate model. Satish et al. [15] presented thermal vibration analysis of orthotropic nanoplates based on nonlocal continuum mechanics.

Also, investigation of the bending and buckling of graphene sheets under thermal loads has attracted attentions in recent years. Zenkour and Sobhy [16] studied the thermal buckling of nanoplates lying on Winkler–Pasternak elastic substrate medium using nonlocal elasticity theory. Sobhy [17] employed a Levy-type solution model to demonstrate the bending response of single-layered graphene sheets subjected to a temperature field as well as external mechanical load. He derived the governing differential equations of the thermo mechanical response on the basis of Eringen's nonlocal elasticity equations incorporated with the two-variable plate theory. Alzahrani et al. [18] analyzed the hygro-thermo-mechanical bending of SLGSs embedded in an elastic medium by considering small scale effect. Sobhy [19,20] investigated on the effect of hygrothermal conditions on the bending of nanoplates using Levy type solution model employing the state-space concept. Also, he reformulated the sinusoidal shear deformation plate theory (SDPT) using the nonlocal differential constitutive relations of Eringen to analyze the bending and vibration of the nanoplates, such as single-layered graphene sheets, subjected to mechanical and thermal loads and resting on two-parameter elastic foundations. Malekzadeh et al. [21] considered small scale effect on the thermal buckling of orthotropic arbitrary straight-sided quadrilateral nanoplates embedded in an elastic medium. Wang et al. [22] reported thermal buckling properties of rectangular nanoplates with small-scale effects. They derived the critical temperatures for the nonlocal Kirchhoff and Mindlin plate theories by nonlocal continuum mechanics. From their work, it can be concluded that the small-scale effects are significant for the thermal buckling properties of nanoplates.

There is just one analytical study on vibration analysis of an eccentric annular graphene sheet by Fadaee and Ilkhani [23]. They used nonlocal thin annular plate to model the problem and solved equation of motion for various boundary conditions by using transitional addition theorem for cylindrical vector wave function.

According to the above literature review, although several attempts by researchers have been made to consider the effects of thermal loads on static and dynamic behaviors of circular and rectangular SLGSs, there is no work concerning an analytical approach for the thermal vibration of a circular SLGS with an eccentric hole. Hence, this paper is the first effort that provides an analytical method for accurate model of free vibration of an eccentric circular SLGS under thermal load.

This article is concerned with an analytical procedure to analyze effects of thermal load and an eccentric circular hole on natural frequencies of a circular SLGS. The eccentric circular hole can be located in various positions with respect to the main circle center. Nonlocal plate theory of Eringen is used to extract equation of motion. Natural frequencies of the SLGS are obtained using the separation of variables method as well as translational addition theorem. Accuracy and stability of the present approach are validated by the literature. From the present study, it can be concluded that, depending on the selection of the temperature change and size and location of the eccentric hole, the stiffness of the SLGS may decrease or increase leading to the decrease or increase in the natural frequencies, respectively.

2. Mathematical formulations

2.1. Nonlocal elasticity model for thermal vibration of the SLGS

According to Fig. 1, consider the arrangement of carbon atoms that are connected together with Covalent bonds, as a circular graphene sheet. With the assumption that some carbon atoms are eliminated in an arbitrary location, a circular graphene sheet with an eccentric circular perforation will be created as shown in Fig. 1. The parameters R_1, R_2, h, ρ and E indicate outer radius, inner (perforation) radius, thickness, density and modulus of elasticity of the graphene sheet, respectively. The distance between centers of circular sheet and circular perforation as eccentricity is shown by ε. To establish mathematical formulations and the analytical procedure, it is required to take two polar coordinates to coincide with the center of circular sheet and the center of its circular perforation as (r_1, θ_1) and (r_2, θ_2), respectively as shown in Fig. 1.

According to the nonlocal theory presented by Eringen [1], behavior of a Hookean solid is represented by the following differential constitutive relation:

$$(1-\mu^2\nabla^2)\boldsymbol{\sigma}^{nl}=\boldsymbol{\sigma}^{l} \tag{1}$$

where $\nabla^2=\left((\partial^2/\partial r^2)+(1/r)(\partial/\partial r)+(1/r^2)(\partial^2/\partial\theta^2)\right)$ is the Laplacian operator, e_0 is constant correlated by material type, $\mu=e_0\,a$ is nonlocal parameter and a is internal characteristic length.