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# Effect of microstructure on vibration characteristics of partially cracked rectangular plates based on a modified couple stress theory

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ABSTRACT

An analytical model for vibration analysis of partially cracked isotropic and functionally graded rectangular plates is presented based on a modified couple stress theory. A single internal material length scale parameter capturing the microstructure is incorporated in the equation of motion of cracked plates derived using the equilibrium principle. Two crack configurations (i) a single partial continuous line crack and (ii) two perpendicular continuous line cracks located at the centre of the plate are considered for analysis. The effect of the single and the two perpendicular partial cracks is incorporated in the form of moment and in-plane forces based on a simplified line spring model. Berger's formulation for the in-plane forces makes the model nonlinear and Galerkin's method is used for its solution. Results for the fundamental frequencies as affected by plate aspect ratio, crack length, internal material length scale parameter and gradient index are presented for various boundary conditions. It is found that the microstructure of the plate affects the fundamental frequencies.



## 1. Introduction

The applications of plate structure are evident in many fields of engineering. A large number of structural components in engineering can be classified as plates. Since plates are often under dynamic loading, it is important to understand their vibration characteristics. Literature shows that the vibration characteristics are affected by the presence of singularities in the form of cracks. Using the classical plate theory, Rice and Levy [1] developed the line spring model for cracked plate with stretching and bending compliances to find stress intensity factors. They also showed the variation of tensile and bending compliances with crack depth. King [2] developed a set of linear algebraic equations to analyse fracture parameters by simplifying coupled integral equations of the line spring model developed by Rice and Levy [1]. Zhao-Jing and Shu-Ho [3] used the simplified model of King [2] and developed an analytical formulation to find the stress intensity factors for a rectangular plate with an inclined crack. Maruyama and Ichinomiya [4] showed the variation of natural frequency with angular orientation of narrow slits for clamped rectangular plates in their experiments. Huang et al. [5] developed a set of new admissible functions in their application of the Ritz method for free vibration analysis of simply supported square plate having arbitrary located through internal crack at various orientations. Khadem and Rezaee [6] established an analytical method for vibration analysis of cracked plates by introducing modified comparison functions for cracked rectangular plate considering bending compliance. They concluded that the natural frequency is affected by the presence of crack at a specific depth and location. Experimental approach has been adopted by Wu and Law [7] and they found that the natural frequencies of thick plates are affected by the orientation of the crack. Vibration and stability of rectangular plate with a through crack were analysed by Stahl and Keer [8]. Solecki [9] gave a solution for bending vibration of rectangular plate with a crack parallel to one of the edges of the plate. Cheng and Reddy [10] used green's functions for infinite and semiinfinite anisotropic thin plates in their work on stress analysis. Studies have also been done where researchers considered crack compliances for bending to study the effect of the crack on lateral deflection and load carrying capacity of columns [11]. Krawczuk et al. [12] developed a finite element model for a plate containing an elastic–plastic through crack. Israr et al. [13,14] used the line spring model developed by Rice and Levy [1] to find an approximate analytical solution for vibrations of isotropic rectangular plates containing a surface crack parallel to one of the edges and located at the centre. They applied the method of multiple scales to obtain the amplitude response for various boundary conditions.

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They also calculated compliance coefficients for bending and tension considering crack depth to plate thickness ratio of 0.6 and concluded that with an increase in crack length the fundamental frequency decreases. Line spring model has also been used by Joshi et al. [15] where the authors presented results for vibration analysis of isotropic plate containing two perpendicular continuous line cracks.

A critical review of recent research on functionally graded material (FGM) plate was presented by Jha et al. [16]. Their work focused on thermoelastic static analysis and vibration and stability analysis of functionally graded plates. Batra and Jin [17] developed a finite element scheme to analyse functionally graded anisotropic plate based on the first order shear deformation theory. Vel and Batra [18] using the power series method presented an exact solution for three-dimensional thermo-elastic deformations of a simply supported functionally graded thick rectangular plate. They compared their results with the classical, first order and third order shear deformation theories for functionally graded plates. Akbari et al. [19] used Meshless Local Petrov–Galerkin method to simulate propagation of a thermoelastic wave through FGM plate and presented results for dynamic displacement and stress fields under transient thermal field. Yin et al. [20] studied bending, buckling and free vibration of FGM plate and formulated a simple and efficient plate element method for functionally graded plates. Rahimabadi et al. [21] presented results for vibration of FGM plates with cutout and crack in thermal environment using an enriched shear flexible 4-noded quadrilateral element with a Heaviside function. Ganapathi et al. [22] studied critical buckling of simply supported functionally graded skew plates subjected to mechanical loads. They formulated a finite element model based on first order shear deformation theory and evaluated results for the critical buckling load as affected by the gradient index, aspect ratio and skew angle. The work of Natarajan et al. [23–26] shows application of finite element method to study static bending, free vibration, mechanical and thermal buckling of intact and cracked FGP. They used 8-noded shear flexible element [23], enriched 4-noded quadrilateral element [24], 3-noded triangular element [25] and NURBS basis functions based iso-geometric finite element method [26] and presented results for natural frequencies of FGM plate as affected by gradient index, through crack length and temperature gradient. Shojaee et al. [27] presented an iso-geometric finite element method for analysis of natural frequencies of thin plate problems of various geometries using NURBS basis function. Reddy [28] presented deflection and stress analysis of FGM plates based on third order shear deformation theory with linear and nonlinear finite element models.

Experimental data reported by various workers indicate that the micro-structure of the plate affects its behaviour. The conventional continuum theory cannot explain the effect of microstructure and hence theories for microstructures need to be developed. Theories for micro-structures include couple stress theory and strain gradient theory. The work of Papargyri-Beskou and Beskos [29] shows the inclusion of strain gradient coefficients and constant in-plane forces in the equation of motion resulting in sixth order partial differential equation. The modified couple stress theory was proposed by Yang et al. [30] in which the couple stress tensor was symmetric and only one internal material length scale parameter was considered. Tsiatas [31] developed a new Kirchhoff plate model for the static analysis of isotropic micro-plates with arbitrary shape based on a modified couple stress theory [30] considering only one material length scale parameter which can capture the size effect. Yin et al. [32] developed a model for the dynamic analysis of intact microscale plates based on the modified couple stress theory in which an internal material length scale parameter was included. They concluded that for an intact plate the fundamental frequency is higher than the classical results thus showing the dependence of size effect on thin plates. Jung et al. [33] developed a model for sigmoid functionally graded material (S-FGM) microplates based on modified couple stress theory coupled with the first order shear deformation theory. They assumed the material properties of functionally graded plate to vary according to two power law distributions. Tadi Beni et al. [34] developed a size dependent equation of motion for functionally graded cylindrical shell using shear deformation model.

To the best of the author's knowledge, literature lacks in the results for vibration analysis of partially cracked plates considering the effect of microstructure. The present work addresses this by proposing a nonclassical Kirchhoff plate analytical model which combines the classical plate theory and the modified couple stress theory. The present model for vibration analysis of cracked plates addresses the following:

1) An isotropic and a general functionally graded rectangular plate are considered for analysis.
2) Two crack configurations (i) a single partial continuous line crack and (ii) two perpendicular continuous line cracks are considered.
3) A single internal material length scale parameter based on a modified couple stress theory is considered in the model which captures the size effect of the microstructure.

The present model is developed by formulating an equation of motion for partially cracked plate considering the effect of internal material length scale parameter. The cracks are in the form of continuous lines and each is parallel to one of the edges of the plate. Such line cracks only affect the stiffness of the plate and not its mass. The plate considered is shown in Fig. 1. It shows two partial cracks perpendicular to each other and located at the

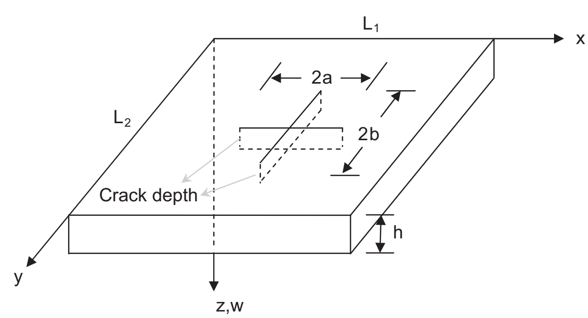


**Fig. 1.** Two perpendicular cracks located at the centre of the FGM plate.

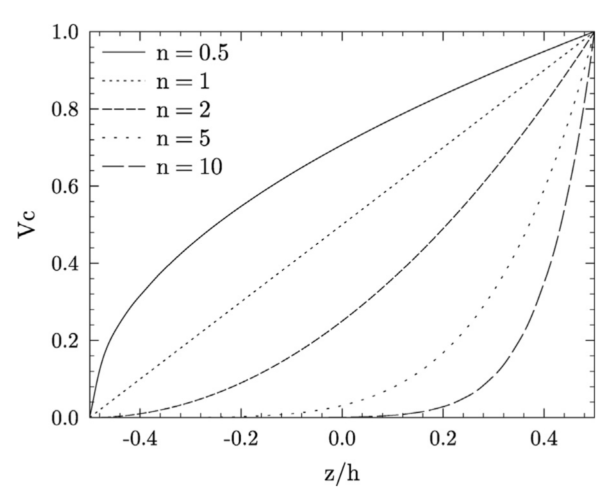


**Fig. 2.** Variation of volume fraction along the thickness of the plate.