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A semi-analytical study on static behavior of thin skew plates on Winkler and Pasternak foundations

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ABSTRACT

This study presents a semi analytical closed-form solution for governing equations of thin skew plates with various combination of clamp, free and simply supports subjected to uniform loading rested on the elastic foundations of Winkler and Pasternak. The governing forth-order partial differential equation (PDE) of two-variable function of deflection, $w(X,Y)$, is defined in Oblique coordinates system. Application of EKM together with the idea of weighted residual technique, converts the forth-order governing equation to two ODEs in terms of X and Y in Oblique coordinates. Both resulted ODEs, are then solved iteratively in a closed-form manner with a very fast convergence. Finally deflection function is obtained. It is shown that some parameters such as angle of skew plate and stiffness of elastic foundation have an important effect on the results. Also it is investigated that shear stresses exist considerably in skew plates comparing to the corresponding rectangular plates. Comparisons of the deflection and stresses at the various points of the plates show very good agreement with results of other analytical and numerical analyses.

1. Introduction

Kerr [1] developed the idea of the well-known Kantorovich method [2] to obtain highly accurate approximate closed-form solution for torsion of prismatic bars with rectangular cross-section. The method employs the novel idea of Kantorovich to reduce the governing partial differential equation of a two-dimensional (2D) elasticity problem to a double set of ordinary differential equations. Since then, the Extended Kantorovich Method (EKM) extensively has been applied for various 2D elasticity problems in Cartesian coordinates system. Among these applications, one can refer to eigenvalue problems [3], buckling [4] and free vibrations [5] of thin rectangular plates, bending of thick rectangular isotropic [6,7] and laminated composite [8] plates and free-edge strength analysis [9]. Most recent EKM based articles include vibration of variable thickness [10] and buckling of symmetrically laminated [11] rectangular plates. Accuracy of the results and rapid convergence of the method together with possibility of obtaining closed-form solutions for ODE systems have been discussed in these articles and others [12]. Finally, a few research consider polar coordinates e.g. using EKM for sector plates [13]. All these applications of EKM, are devoted and restricted to the problems in the Cartesian and polar coordinate systems. The authors of the present paper, for the first time applied EKM in Oblique coordinate system for bending of skew plates under clamp boundary conditions without considering foundations and stress analysis [14]. Based on the other solution methods, several research have studied bending, buckling, vibration and other analysis for skew plates in term of Oblique coordinate system [15–20]. Winkler and Pasternak foundations are considered in the design of structures rested on elastic mediums. Winkler model considers the foundation as a series of springs which do not have any interaction with each other. More advanced models like Pasternak simulate the coupling between these springs too [21,22].

This study aims to examine the applicability of the EKM to obtain highly accurate approximate closed-form solutions for 2D elasticity problems in Oblique coordinate system. Applying Extended Kantorovich Method (EKM) with the aid of a weighted residual technique (Galerkin method), the governing PDE, is converted to two uncoupled ordinary differential equations (ODE) of $f(X)$ and $g(Y)$. Then an initial guess function is considered for one of those functions to obtain the constants of the ODE of the other function. After solving the first ODE, constants of the second ODE are achieved. Then the second ODE is solved for obtaining the first ODE's constants. These iterations continues unless a good convegence is achieved. In every iteration step, exact closed-form solutions are obtained for two ODE systems. Deflection and stress analysis of thin isotropic skew plates with a various combinations of clamp, free and simply supports subjected to uniform loading and resting on the Winkler and

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Pasternak foundations as Fig. 1, is considered. Comparisons of the deflections and stresses at the various points of the skew plate show very good agreement with the results of other valid literatures and FEM analysis of ANSYS code. Discussions reveals the existence of shear stresses in skew plates comparing to the corresponding rectangular plates.

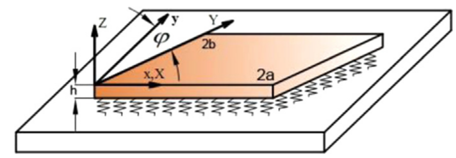

Fig. 1. Skew plate in Oblique coordinate (*X,Y*) resting on the elastic foundations with the stiffness of *k*.

2. Governing equations

If no axial force exists, differential equation of motion is expressed as [23]

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+q(x,y)-k_0 w(x,y)+k_1\nabla^2 w(x,y)=0 \tag{1}$$

and in terms of w as

$$\nabla^4 w(x,y)=\frac{q(x,y)+k_0 w(x,y)-k_1\nabla^2 w(x,y)}{D} \tag{2}$$

Eq. (2) is the governing equation for a thin plate, in which $w(x,y)$ is the deflection function, q is the applied distributed load, D is flexural rigidity for isotropic plates, k_0 and k_1 are the stiffness of Winkler and Pasternak foundation respectively. Having just Winkler foundation it is enough to equal k_1 to zero. Now, consider a thin skew plate with dimensions of $2a\times 2b$ as Fig. 1. For a clamp–support, deflection (w) and its first derivative with respect to the normal direction of the boundary must be vanished. For a simply-support, deflection and its second derivative with respect to the normal direction of the boundary must be vanished. Considering Fig. 1, for example for SSSC boundary conditions (S and C represent simply and clamp respectively) we have

$$w=d^2w/dx^2=0 \text{ for } x=0,\ x=2a$$
$$w=d^2w/dy^2=0 \text{ for } y=0 \text{ and } w=dw/dy=0 \text{ for } y=2b \tag{3}$$

Governing Eqs. (1) and (2) must be converted from Cartesian coordinates system (x,y) to Oblique coordinates system (X,Y) as it is shown in Fig. 1. The relations between Cartesian(x,y) and Oblique(X,Y) are

$$X=x-y\tan\varphi \text{ and } Y=y/\cos\varphi \tag{4}$$

Consequently operator ∇^2 in Cartesian coordinates could be converted to Oblique coordinates as $\overline{\nabla}^2$

$$\overline{\nabla}^2=\frac{1}{\cos\varphi}\left(\frac{\partial^2}{\partial X^2}-2\sin\varphi\frac{\partial^2}{\partial X\partial Y}+\frac{\partial^2}{\partial Y^2}\right) \tag{5}$$

Also, ∇^4 becomes

$$\overline{\nabla}^4=\frac{1}{\cos^4\varphi}\left\{\frac{\partial^4}{\partial X^4}+2\left(1+2\sin^2\varphi\right)\frac{\partial^4}{\partial X^2\partial Y^2}-4\sin\varphi\left(\frac{\partial^4}{\partial X^3\partial Y}+\frac{\partial^4}{\partial X\partial Y^3}\right)+\frac{\partial^4}{\partial Y^4}\right\} \tag{6}$$

For the governing Eq. (2) in Oblique coordinates system we have

$$D\overline{\nabla}^4 w(X,Y)+k_0 w(X,Y)-k_1\overline{\nabla}^2 w(X,Y)=q(X,Y) \tag{7}$$

3. Iterative solution by EKM

According to the Extended Kantorovich Method (EKM) [1], the two-variable-function of the plate deflection, *w(X,Y)* is assumed as multiplication of different single variable functions as

$$w_{ij}(X,Y)\cong f_i(X)\cdot g_j(Y) \tag{8}$$

where $f_i(X)$ and $g_j(Y)$ are unknown functions to be determined and subscripts i and j denote number of iterations. Using Eq. (6), expanding of Eq. (7) is

$$D\overline{\nabla}^4 w+k_0 w(X,Y)-k_1\overline{\nabla}^2 w(X,Y)=q(X,Y)$$
$$\frac{D}{\cos^4\varphi}\left\{\frac{\partial^4 w}{\partial X^4}+2\left(1+2\sin^2\varphi\right)\frac{\partial^4 w}{\partial X^2\partial Y^2}-4\sin\varphi\left(\frac{\partial^4 w}{\partial X^3\partial Y}+\frac{\partial^4 w}{\partial X\partial Y^3}\right)\frac{\partial^4 w}{\partial Y^4}\right\}+k_0 w-\frac{k_1}{\cos\varphi}\left(\frac{\partial^2 w}{\partial X^2}-2\sin\varphi\frac{\partial^2 w}{\partial X\partial Y}+\frac{\partial^2 w}{\partial Y^2}\right)=q(X,Y) \tag{9}$$

By considering assumption of Eq. (8), we have

$$(\cos^4\varphi)\cdot\overline{\nabla}^4(f(X).g_0(Y))+\tfrac{k_0\cos^4\varphi}{D}(f(X).g_0(Y))-\tfrac{k_1\cos^3\varphi}{D}\left(\tfrac{\partial^2(f(X).g_0(Y))}{\partial X^2}-2\sin\varphi\frac{\partial^2(f(X).g_0(Y))}{\partial X\partial Y}+\frac{\partial^2(f(X).g_0(Y))}{\partial Y^2}\right)$$
$$=\left\{g_0(Y)\frac{d^4 f(X)}{dX^4}+2\left(1+2\sin^2\varphi\right)\frac{d^2 g_0(Y)}{dY^2}\frac{d^2 f(X)}{dX^2}-4\sin\varphi\left(\frac{dg_0(Y)}{dY}\frac{d^3 f(X)}{dX^3}+\frac{d^3 g_0(Y)}{dY^3}\frac{df(X)}{dX}\right)+\frac{d^4 g_0(Y)}{dY^4}f(X)\right\}$$
$$+\frac{k_0\cos^4\varphi}{D}(f(X)\cdot g_0(Y))-\frac{k_1\cos^3\varphi}{D}\left(g_0(Y)\frac{d^2 f(X)}{dX^2}-2\sin\varphi\frac{dg_0(Y)}{dY}\frac{df(X)}{dX}+f(X)\frac{d^2 g_0(Y)}{dY^2}\right)=\frac{\cos^4\varphi}{D}q(X,Y) \tag{10}$$

For Eq. (7), according to the Galerkin weighted residual method, we have [13]

$$\int_0^{2a}\int_0^{2b}(D\overline{\nabla}^4 w-q+k_0 w-k_1\overline{\nabla}^2 w)\delta w\,dX\,dY=0 \tag{11}$$

Now, for a prescribed function of $g_j(Y)$, $j=0$ and referred to Eq. (8), δw becomes

$$\delta w=g_0(Y)\cdot\delta f_i \tag{12}$$

Substitution of Eq. (8) into Eq. (11) in conjunction with Eq. (12) leads to

$$\int_0^{2a}\left[\int_0^{2b}\left(D\overline{\nabla}^4(f_i\cdot g_0)-q+k_0(f_i\cdot g_0)-k_1\overline{\nabla}^2(f_i\cdot g_0)\right)g_0\,dY\right]\delta f_i\,dX=0 \tag{13}$$

Based on the existing rules in the Variational principle, Eq. (13) is satisfied if the expression in the bracket is vanished

$$\int_0^{2b}\left(D\overline{\nabla}^4(f_i\cdot g_0)-q+k_0(f_i\cdot g_0)-k_1\overline{\nabla}^2(f_i\cdot g_0)\right)g_0\,dY=0 \tag{14}$$