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Contact problem for a functionally graded layer indented by a moving punch

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ABSTRACT

In this study moving contact problem for a rigid cylindrical punch and a functionally graded layer is considered. The punch subjected to concentrated normal force, and moves steadily with a constant subsonic velocity on the boundary. Poisson's ratio is taken as constant, and both the elasticity modulus and the mass density are assumed to vary exponentially in depth direction. By using Fourier transform and boundary conditions, the governing equations are reduced to a Cauchy singular integral equation. The numerical solution of the singular integral equation is obtained by using Gauss–Chebyshev integration formulas. Numerical results for the contact area, the contact stress and the normal stresses are given. This study is limited in that the elasticity modulus and the mass density vary with the same function. However, it is the first attempt to investigate the moving contact problem with FGMs.

1. Introduction

Functionally graded materials (FGMs) are a new kind of inhomogeneous composites whose material properties are designed for specific function. The concept is to form a composite material by varying the microstructure from one material to another with a particular gradient. Some potential application of FGMs involve contact problems which include bearings, gears, cams, gas turbines, brake disks and other automotive components.

The contact mechanics of FGMs have received increasing research efforts during the past four decades. Bakirtas [1] studied the contact problem of a non-homogeneous elastic half-space indented by a rigid punch whose shape can be flat or circular. Giannakopoulos and Suresh [2,3] presented the analytical and computational investigations of an axisymmetric problems of a graded elastic medium subjected to a point force or a rigid indenter. The elastic modulus is assumed to vary in depth according to a power law or an exponential function. Giannakopoulos and Pallot [4] presented the closed form analytical solutions of two-dimensional normal, sliding and rolling type of contact of a rigid cylinder on an elastic graded substrate. It was demonstrated that power law gradation can be very beneficial in the design of strong and wear resistance sliding surface.

Problems involving the graded coatings bonded to a homogeneous substrate are widely examined by several researchers. Guler and Erdogan [5,6] considered the analytical solution of the sliding contact problem in FG coatings by assuming an exponential variation of the elastic modulus. The loading was provided by a sliding rigid stamp whose shape can be triangular, cylindrical or semi-circular. A linear multi-layered model for the two-dimensional frictional and frictionless contact problems is presented using the Fourier transforms in the studies of Ke and Wang [7,8]. They divided the FGM into several sub-layers assuming the shear modulus is a linear function. Liu et al.[9] and Liu and Wang [10] solved the axisymmetric frictionless contact problem of a FG coated half-space by applying the Hankel transform technique. They calculated the axisymmetric contact of the flat circular, spherical and conical indenters. The stress analysis of a three-layer coating structure with an intermediate layer made of FGMs was solved by applying the linear multi-layered model in the study of Yang and Ke [11]. Chen and Chen[12] considered a contact model between a rigid punch and a homogeneous half-space coated with a linear graded layer. Their study demonstrated that the stress singularity depends only on the friction coefficient and surface Poisson's ratio. Guler et al. [13] studied analytical and numerical solution of a contact problem of thin films bonded to graded coatings.

When two bodies contact each other, the applied loads cause the bodies to deform and the initial contact area changes. This type of contact is termed a receding contact, and the contact area is the primary unknown in the problem. Receding contact problems between a FG layer and a homogeneous substrate/layer are studied by [14–17]. El-Borgi et al. [14] and Rhimi et al. [15] studied the frictionless receding contact plane and axisymmetric problems between a homogeneous substrate and an elastic FG layer subjected to finite normal tractions. El-Borgi et al. [16] extended this

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problem considering the friction. In a recent study, Jie and Xing [17] studied the double receding contact problem between a FG layer and an elastic layer neglecting friction.

Contact problem of a rigid punch and a single FG layer resting on a Winkler foundation or a rigid substrate is investigated by Comez [18] and Choi [19]. Kashtalyan and Menshykova [20] analyzed the elastic deformation of a FGM coating/substrate system of finite thickness within the framework of three-dimensional elasticity theory. Dag et al. [21] presented the analytical and computational investigations of a frictional contact between a rigid punch and a FG substrate that possess material property in the lateral direction. Thermoelastic contact problems of FGMs are studied by Barik et al. [22], Ke and Wang [23] and Chen and Chen [24].

There are problems that arise in practice in which the speed of one body relative to other is quite large, and we therefore need to investigate whether it is necessary to take the dynamic character of the problem into account Galin [25]. Eringen [26] investigated the indentation of a half-space by a rigid frictionless moving punch. Zhou et al. [27] studied a contact problem for orthotropic half plane indented by a moving rigid punch with various punch profiles in the presence of friction.

Although there is much research available in the literature related to the contact problem of FGMs, in these studies the effect of the moving velocity has not been investigated. In this study contact problem for a moving rigid cylindrical punch and a functionally graded layer is considered. The punch subjected to concentrated normal force, and moves steadily with a constant subsonic velocity on the boundary. Poisson's ratio is taken as constant, and both the elasticity modulus and the mass density are assumed to vary exponentially in depth direction. The problem is reduced to a singular integral equation of the first kind, in which the contact stress and the contact area are the unknowns, and it is treated using Fourier transforms and the boundary conditions for the problem. The numerical solution of the singular integral equation is obtained by using Gauss–Chebyshev integration formulas. Numerical results for the contact area, the contact stress and the normal stresses are given. This study is limited in that the elasticity modulus and the mass density vary with the same function.

2. Formulation of the problem

Consider the plane strain contact problem in Fig. 1. A FG layer of a thickness h is firmly attached to a rigid substrate. The loading is provided by a rigid punch with radius R subjected to concentrated normal force P. The punch moves on the layer in x_1 direction at a constant velocity V. Poisson's ratio is taken as constant, and both the shear modulus and the mass density are assumed to vary exponentially in depth direction as follows

$$\mu(y)=\mu_0 e^{\gamma\, y}, \quad \rho(y)=\rho_0 e^{\gamma\, y} \tag{1}$$

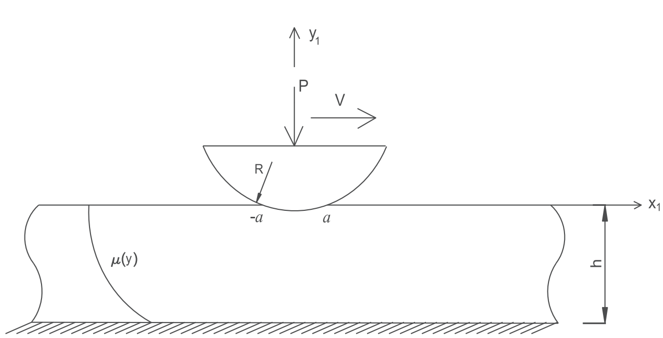

Fig. 1. Geometry of the moving punch problem.

where $\mu_0$and ρ_0 are the shear modulus and mass density on the top surface of the layer respectively. γ is a constant characterizing the material inhomogeneity.

In the absence of body forces the wave equations of elasto-dynamics can be written as

$$\frac{\partial \sigma_x}{\partial x_1}+\frac{\partial \tau_{xy}}{\partial y_1}=\rho(y)\frac{\partial^2 u}{\partial t^2} \tag{2a}$$

$$\frac{\partial \tau_{yx}}{\partial x_1}+\frac{\partial \sigma_y}{\partial y_1}=\rho(y)\frac{\partial^2 v}{\partial t^2} \tag{2b}$$

where u and v are the x_1- and y_1-components of the displacement vector and t denotes the time variable.

Assuming that the FG layer is isotropic at every point, for the plane contact problem Hooke's law can be written as follows:

$$\sigma_x(x,y)=2\mu(y)\left[\frac{(1-\nu)}{1-2\nu}\frac{\partial u}{\partial x_1}+\frac{\nu}{1-2\nu}\frac{\partial v}{\partial y_1}\right] \tag{3a}$$

$$\sigma_y(x,y)=2\mu(y)\left[\frac{\nu}{1-2\nu}\frac{\partial u}{\partial x_1}+\frac{(1-\nu)}{1-2\nu}\frac{\partial v}{\partial y_1}\right] \tag{3b}$$

$$\tau_{xy}=\mu(y)\left[\frac{\partial u}{\partial y_1}+\frac{\partial v}{\partial x_1}\right] \tag{3c}$$

where ν represents Poisson's ratio.

Due to constant speed, it is tractable to introduce Galilean transformation [25]

$$x=x_1-Vt \quad y=y_1 \tag{4}$$

The dynamic contact problem can be reduced to the steady contact problem after introducing Eq. (4).

Substituting Eq. (4) into the Eq. (2) and using Hooke's law [3], the equations of equilibrium in terms of the $u(x,y)$ and $v(x,y)$ are obtained as follows

$$(-1+2\nu)\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 v}{\partial x\partial y}+\left(2(-1+\nu)-(-1+2\nu)\frac{V^2}{c_T^2}\right)\frac{\partial^2 u}{\partial x^2}$$
$$+\gamma\left((-1+2\nu)\frac{\partial u}{\partial y}-\nu\frac{\partial v}{\partial x}\right)=0 \tag{5a}$$

$$+2(-1+\nu)\frac{\partial^2 v}{\partial y^2}-\frac{\partial^2 u}{\partial x\partial y}+(-1+2\nu)\left(1-3\frac{V^2}{c_L^2}\right)\frac{\partial^2 v}{\partial x^2}$$
$$+2\gamma\left(-\nu\frac{\partial u}{\partial x}+(-1+\nu)\frac{\partial v}{\partial y}\right)=0 \tag{5b}$$

where c_L and c_T are the speed of propagation of longitudinal and transverse waves in the layer respectively.

$$c_T=\sqrt{\frac{\mu_0}{\rho}}\quad c_L=\sqrt{\frac{2\mu_0+\lambda_0}{\rho}} \tag{6}$$

Assuming that $x=0$ is a plane symmetry, it is sufficient to consider the problem in the region $0\le x<\infty$ only. Using the symmetric property of the problem and Fourier transform technique, the following expressions may be written

$$u(x,y)=\frac{2}{\pi}\int_0^\infty \tilde{u}(\alpha,y)\sin(\alpha x)d\alpha \tag{7a}$$

$$v(x,y)=\frac{2}{\pi}\int_0^\infty \tilde{v}(\alpha,y)\cos(\alpha x)d\alpha \tag{7b}$$

where $\tilde{u}(\alpha,y)$ and $\tilde{v}(\alpha,y)$ are Fourier transforms of $u(x,y)$ and $v(x,y)$, respectively.