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Helical buckling of a thin rod with connectors constrained in a torus

Wenjun Huang *, Deli Gao *

College of Petroleum Engineering, China University of Petroleum-Beijing, 102249 China

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ABSTRACT

Kinds of tubular strings are widely applied to oil exploration & development. In the preceding studies, theses tubular strings are usually taken as homogeneous thin rods without connectors constrained in a straight cylinder or torus. Recent studies have indicated that the effect of connectors on the tubular strings is ignorable. On the basis of the previous studies, the paper provides an analytical model describing the helical buckling behaviors of a rod with connectors constrained in a torus. The new model is solved based on beam-column model and minimum potential energy theory. There are three contact cases between the rod and the torus, namely no contact, point contact and wrap contact. The deflections of the rod and critical conditions between different contact cases are calculated. The results show that the deflections and critical conditions are quite different when the equivalent torus curvature radius changes. The effects of equivalent torus curvature radius, rod buckling and connectors on bending moment and contact force are analyzed. The results show that connector is an important factor which should not be neglected.

1. Introduction

Tubular strings such as drill string, casing, riser and tubing are playing an important role in petroleum exploration and development. For example, the drill string can transmit ground power to the drill bit to break underground rocks (Jorge and Sampaio [1]) in Fig. 1. The drill string is usually taken as a thin elastic rod, and the wellbores in the vertical, horizontal and building sections are taken as straight cylinders and tori which constrain the lateral deflection of the rod.

For a rod constrained in a horizontal cylinder or torus, the rod lies on the bottom of the cylinder/torus and keeps its initial configuration when the axial compression on the rod is rather small. When the axial compression exceeds a certain value, the rod buckles in a sinusoidal configuration in which the rod snakes along the lower surface of the cylinder/torus. With the further increase of the axial compression, the rod achieves a helical buckling configuration in which the rod approximately forms a helix spiraling around the inner surface of the cylinder and torus (Wicks et al. [2]). Therefore, the buckling state of a rod is usually divided into initial configuration, sinusoidal buckling and helical buckling. The critical axial compressions from initial configuration to sinusoidal buckling (Paslay and Bogy [3], Mitchell [4]) and further to helical buckling in vertical (Chen et al. [5]), straight inclined and curved wellbores(Gao et al. [6], Gao et al. [7], Liu [8], Huang et al. [9]) have been fully studied. And the post buckling behaviors of the tubular string for sinusoidal buckling and helical buckling are also deduced with buckling differential equation (Mitchell [10]) and energy method (Gao et al. [6,7] and Liu [8]). Researches show that boundary conditions (Gao et al. [11]) and connectors on the tubular string (Mitchell [12–16], Gao et al. [17], Huang et al. [18,19]) play an important role in tubular string buckling. Meanwhile, tubular string buckling affects bending moment (Lubinski et al. [20]) and axial force transfer (Mitchell [21,22], Wu et al. [23]) a lot. Cunha [33] presented the reason for the conflicting results about the critical helical buckling loads. Hajianmaleki and Daily [34] reviewed the progress of the studies on the tubular string buckling and pointed the areas that need more research efforts. The above research results have been widely applied to engineering operations.

In this paper, our research is mainly focused on the effect of connectors on tubular string buckling. Previous studies (Duman et al. [27], Mitchell et al. [28,30–31], Weltzin et al. [29]) show that the effect of connectors on drill string buckling is non-ignorable. Connectors are assumed to distribute discretely along the rod and the diameters of connectors are larger than that of the rod body. Therefore, some portions of the rod lose

* Corresponding authors. Tel.: +86 15910852018.
E-mail addresses: huangwenjun1986@126.com (W. Huang), gaodeli@cast.org.cn (D. Gao).

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contact with the cylinder/torus for the existence of connectors. Then there are three contact cases between the rod and the cylinder/torus: no contact, point contact and wrap contact. No contact means that the rod suspends between connectors and is not in contact with the cylinder/torus; point contact means that the rod is in contact with the cylinder/torus at a single point; wrap contact means that a segment of the rod is in continuous contact with the cylinder/torus.

Lubinski [24] studied the two-dimensional buckling of a rod with connectors in tension constrained in a torus under no contact, point contact and wrap contact cases, and Paslay and Cernocky [25] studied the two-dimensional buckling of a rod with connectors in compression constrained in a torus under no contact, point contact and wrap contact cases. In fact, with the increase of the axial compression, the two dimensional deflection of the rod becomes unstable and the rod buckles into three dimensional deflection. Mitchell [13–16] studied the sinusoidal buckling of the drill string constrained in a horizontal cylinder and torus under no contact case, and the helical buckling constrained in a vertical cylinder under no contact case. Gao et al. [17] studied the critical conditions from initial configuration to sinusoidal configurations constrained in a horizontal cylinder under no contact, point contact and wrap contact. Huang and Gao [18,19] studied the sinusoidal and helical buckling of the drill string constrained in a horizontal cylinder under no contact, point contact and wrap contact cases. Wei et al. [32] studied the sinusoidal buckling of a thin rod with connectors constrained in a torus.

In this paper, the helical buckling of a rod with connectors constrained in a torus is studied. The deflections of the rod under no contact and point contact cases are analyzed, and the critical conditions from no contact to point contact and from point contact to wrap contact are given. The effects of the equivalent torus curvature radius, rod buckling and connectors on bending moment and contact force are also studied.

2. Backgrounds

Fig. 1 presents a typical example of a horizontal well trajectory. The whole trajectory includes three parts: vertical section, building section and horizontal section. The vertical and horizontal sections are seen as straight lines and the building section is seen as an arc. In actual drilling

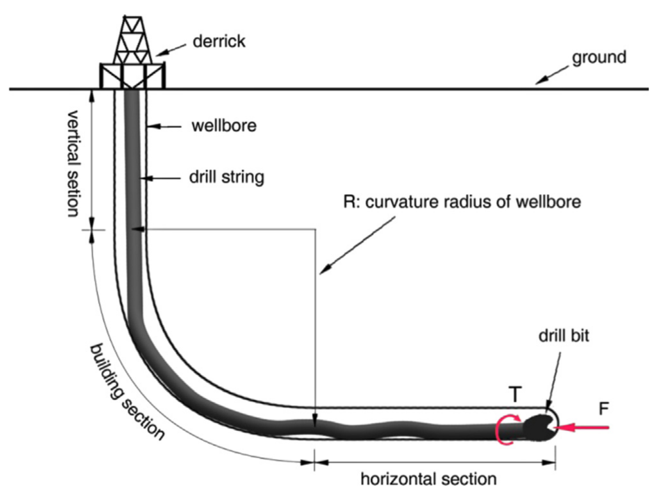

Fig. 1. Drill string in the wellbore.

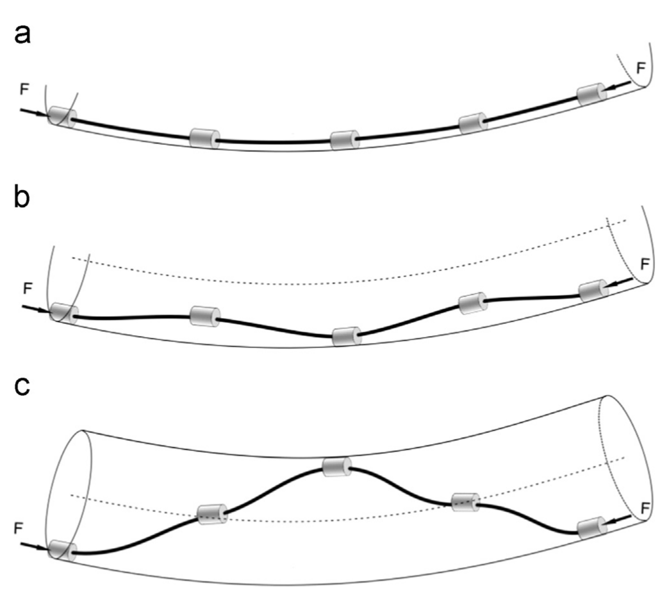

Fig. 2. Buckling state of a rod with connectors constrained in a torus: (a) initial plane configuration, (b) sinusoidal buckling and (c) helical buckling.