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Solutions for behavior of a functionally graded thick-walled tube subjected to mechanical and thermal loads

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ABSTRACT

For the problem of a functionally graded thick-walled tube subjected to internal pressure, we have already presented the elasticity solution based on the Voigt method with the assumption of a uniform strain field within the representative volume element. This paper discusses the thermoelastic problem of the functionally graded thick-walled tube subjected to both axisymmetric mechanical and thermal loads, and gives the solution in terms of volume fractions of constituents. We assume that the tube consists of two linear elastic constituents and the volume fraction of one phase is a power function varied in the radial direction. The theoretical solutions of the displacement and the stresses are presented under the assumption of a uniform strain field within the representative volume element. Comparisons of the theoretical solutions and the finite element analysis demonstrate the validity of the assumption. Based on the relation of the volume average stresses of constituents and the macroscopic stresses of the composite material in micromechanics, the present method can avoid the assumption of the distribution regularities of unknown overall material parameters appeared in existing papers. Further, the present method is valid for the materials with different Poisson's ratios of constituents. The effects of the volume fraction, the ratio of two thermal expansion coefficients and the ratio of two thermal conductivities on the displacement and stresses are systematically studied.

1. Introduction

Functionally graded materials (FGMs) are composite materials formed of two or more constituent phases with a continuously variable composition. FGMs have a lot of advantages that make them attractive in potential applications, including a potential reduction of in-plane and transverse through-the-thickness stresses, an improved residual stress distribution, enhanced thermal properties, higher fracture toughness, and reduced stress intensity factors [1].

In the last two decades, FGMs have been widely used in engineering applications, particularly in high-temperature environment, and power transmission equipment. The functionally graded thick-walled tube is a kind of typical structures such as pressure vessels and cylinders that are utilized in reserving or transferring chemical gas, oil, etc. Thus the thermoelastic problems of thick-walled tubes have been studied by many researchers. Some researchers presented an exact solution for thermal stresses of FGM cylinders [2] and spheres [3] whose Young's modulus and thermal expansion coefficient vary linearly with the radius. However, the linear function assumption is not sufficient for describing more complicated cases. To capture Young's modulus and thermal expansion coefficient of the FGM thick-walled tube more precisely, some researchers [4–18] proposed another assumption of Young's modulus such as the form of $E(r) = E_0 r^{m_1}$ (E_0 and m_1 are material constants, r is the radial coordinate) and thermal expansion coefficient such as the form of $\alpha(r) = \alpha_0 r^{m_2}$ (α_0 and m_2 are material constants). For convenience, thermal conductivity was mostly assumed in the form of $k(r) = k_0 r^{m_3}$ (k_0 and m_3 are material constants). Furthermore, Ozturk and Gulgec [19] proposed the three variable controlled material properties such as Young's modulus in the form of $E(r) = E_0\left[1 - n_E (r/b)^{k_E}\right]$ of (E_0, n_E and k_E are material constants, b is the outer radius), thermal expansion coefficient in the form of $\alpha(r) = \alpha_0\left[1 - n_\alpha (r/b)^{k_\alpha}\right]$ (α_0 , n_α and k_α are material constants) and similar form of the thermal conductivity.

In the above-mentioned works, varying material properties are usually treated as specific gradient variation such as linear form or power-law form. Nevertheless, it is difficult to satisfy the engineering manufacture in practice. Hence, authors [20,21] proposed a new method which assuming the material properties in terms of the volume fraction of one phase such as Young's modulus in the form

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Nomenclature

a	inner radius
b	outer radius
r	radial coordinate
$c(r)$	volume fraction of material A
c_0, k, n	material parameters
p_a, p_b	internal and external pressures
C_{ij} $(i, j=1, 2)$	constant thermal parameters
f_j $(j=1, 2)$	constants on the inner and outer radii
$k(r)$	thermal conductivity
k_i $(i=1, 2)$	thermal conductivity of the component
$T(r)$	temperature variation
C_i $(i=0, 1, 2)$	constants in temperature function
α_i $(i=1, 2)$	thermal expansion coefficient of the component
λ_i, μ_i $(i=1, 2)$	Lamé constants of the component
E_i, ν_i $(i=1, 2)$	Young's modulus and Poisson's ratio of the component
$\varepsilon_r^{(i)}$, $\varepsilon_\theta^{(i)}$	radial and circumferential strains of the component
$\sigma_r^{(i)}$, $\sigma_\theta^{(i)}$, $\sigma_z^{(i)}$	radial, circumferential and axial stresses of the component
σ_r, σ_θ,σ_z	average radial, circumferential and axial stresses of the tube
u	radial displacement
x	a new variable about r
F	hypergeometric function
α, β, δ	coefficients in hypergeometric function
B_i $(i=0, 1, 2)$	intergral constants
$\bar{r}$	non-dimensional radial coordinate
$\bar{a}$, $\bar{b}$	non-dimensional inner and outer radii
$\bar{u}$	non-dimensional radial displacement
$\bar{\sigma}_{ij}$	non-dimensional stresses

of $E(r)=E_cV_c+E_mV_m$ (E_c and E_m are material constants, V_c and V_m are volume fractions) and the thermal expansion coefficient in the form of $\alpha(r)=\alpha_c V_c+\ \alpha_m V_m$ (α_c andα_m are material constants). Though some researchers [22,23] considered the variations of both Young's modulus and the thermal expansion coefficient by using the Mori–Tanaka method, they did not obtain the theoretical solutions due to the complicated calculation of the equilibrium equation presented in their work.

Recently, the Carrera Unified Formulation (CUF), which was developed by Carrera for multi-layered structures [24–28], is extended to also account for functionally graded shells under mechanical and thermal loadings. The Principle of Virtual Displacements (PVD) has been proposed in [29] and the extension to Reissner's Mixed Variational Theorem (RMVT) has been given in [30]. The thermo-mechanical bending problem of functionally graded plates has already been proposed in [31]. Considering the temperature as an external load [32], the static response of functionally graded shells is studied and the governing equations are derived from the Principle of Virtual Displacements.

In most actual systems, the overall elastic modulus, the thermal expansion coefficient, and the thermal conductivity of the FGM tube cannot be found directly, however, they can be obtained in terms of their properties of constituents and the volume fractions in a certain regulation.

In this paper, we first define a volume fraction varied radially rather than the assumption of Young's modulus, thermal expansion coefficient and thermal conduction coefficient of the FGM tube. Using the elasticity solution in [33], the thermoelastic behavior of the functionally graded thick-walled tube subjected to axisymmetric mechanical and thermal loads is investigated in this work. In Section 2 the basic equations of the FGM long tube and the analysis of thermoelastic mechanical behaviors of the tube are described. Section 3 gives the FEM results and discusses the effect of parameter n, the ratio of two thermal expansion coefficients and the ratio of two thermal conductivities. Conclusions are given in Section 4.

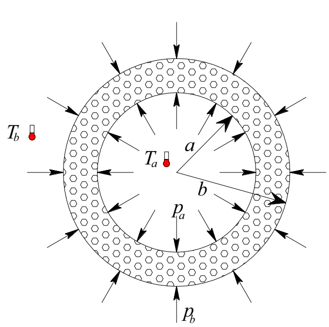

Fig. 1. The cross-sectional contour of a long FGM tube subjected to mechanical and thermal loads.

2. Theoretical analysis

A state of axial symmetry is considered in the problem of a FGM thick-walled tube subjected to axisymmetric mechanical and thermal loads on its inner and outer surfaces (Fig. 1). Cylindrical polar coordinates (r, θ, z) are used and the inner and outer radii of the thick-walled tube are designated as a and b, respectively.

The tube consists of two linear elastic materials A and B, and the volume fraction $c(r) \in [0,\ 1]$ of material A is given by

$$c(r)=c_0\left[1-k(r/b)^n\right] \tag{1}$$

where, r is the radius, c_0, k and n are the material parameters.

2.1. Heat transfer

To obtain desired thermal stresses in the FGM long tube, it is natural to first determine temperature distribution in the thick-wall tube. To this end, we need to consider a steady-state heat conduction problem without internal heat source. The heat equation for steady-state heat conduction with no heat source reads

$$\frac{1}{r}\left[rk(r)T'(r)\right]'=0 \tag{2}$$

and the boundary conditions at the inner and outer surfaces are

$$C_{11}T(a)+C_{12}T'(a)=f_1$$
$$C_{21}T(b)+C_{22}T'(b)=f_2 \tag{3}$$

where the prime denotes the derivative with respect to r, $T(r)$ is the radial temperature variation between the tube and the ambient condition, $k(r)$ is the thermal conductivity, C_{ij} $(i, j=1, 2)$ are the constant thermal parameters relative to the conduction and convection coefficients, and f_j $(j=1, 2)$ are known constants on the inner and outer radii, respectively.