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Integrated orthogonal polynomials based spectral collocation method for vibration analysis of coupled laminated shell structures

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ABSTRACT

In this paper, the spectral collocation method based on integrated orthogonal polynomials is applied to the free vibration analysis of coupled axisymmetric laminated shell structures with arbitrary elastic-support boundary conditions (BCs). The coupled shell structure is firstly divided into its multiple components (i.e. the cone, cylinder, sphere and annular plate) at the location of junction in the meridional direction. Then by applying Hamilton's principle, the equations of motion for all the individual shell segments are derived on the basis of the first-order shear deformation theory. Instead of adopting conventional differentiation scheme, an integration technique is used to each individual segment which leads to a set of algebraic equations. These shell segments are further coupled together by matching all of the required displacement and force continuous conditions at the interface. The remaining elastic-support boundary conditions are employed at the ends of the coupled shells. Accuracy and efficiency of the proposed numerical method are explored through a series of free vibration analysis of joined and stepped shell structures, and the results are compared with those available solutions in open literature. Furthermore, the frequency parameters and mode shapes of selected coupled shell structures including cylindrical–spherical, coupled conical and stepped shells are presented to reveal their geometry- and BC-dependent free vibration characteristics.

1. Introduction

Many structures in industrial applications are actually the combinations of basic structural elements, such as beams, plates and shells. Among these coupled structures, the coupled shells of revolution are typical structural components and have widespread practical applications in civil, mechanical, marine and aerospace engineering. Since these shells are often under dynamic excitations, the research on their mechanical behavior is of great significance for providing optimal design and avoiding resonant vibration responses. The vibration analysis of shell structures has always been one of the most prominent challenges in the field of solid mechanics for many years. According to the monographs by Leissa [1], Reddy [2], Quta [3] and other literature survey articles [4,5], most of the previous research efforts were restricted to the vibrations of elementary shell configurations (e.g., cylinder, cone and sphere) rather than coupled shells. Furthermore, it appears that most of the early studies on the vibration of shells were limited to the ones with classical boundary conditions, such as simply-supported, free and clamped edges. However, the shells with non-classical boundaries such as elastically constrained are frequently encountered in real-world [6] and should be received more attention. In addition, there has been a continuous increase of the application of fiber-reinforced multi-layer composite materials in airplanes, ships and vehicles owing to their merits such as lightweight, high strength and stiffness ratios, and corrosion resistance. Hence, developing a unified and efficient numerical method for a good understanding of the vibration behaviors of the coupled laminated shells with arbitrary elastic-support boundary conditions is crucial to the design of mechanical systems involving these structures.

Depending upon the geometry of the shell, previous research on the free vibrations of coupled shells can be generally categorized into three main groups, namely coupled singly-curved (cylindrical, conical and annular) shells, singly-curved and doubly-curved (spherical, etc.) shell combinations and stepped shells. In order to particularly focus on the features of the present paper, a review of recent works related to the vibration problems of coupled shell structures, including the coupled conical, coupled cylindrical–spherical and stepped shells, is stated below briefly.

The vibration of cylindrical–conical shells was studied by Patel et al. [7], Efraim and Eisenberger [8], Caresta and Kessissoglou [9],

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Kang [10] and Ma et al. [11]. Vibration characteristic of cylindrical–plate or conical–plate shell was investigated by many researchers [12–14]. The studies concerning the vibration behavior of the coupled cylindrical–spherical shell have also been performed, among them the important papers of [15–18] are of particular interest. As for the stepped shells, a few works on the free vibrations have been devoted by Qu et al. [6,19], Xiang and Zhang [20], Hang et al. [21], and Duan et al. [22]. Different shell theories have been applied to solve the vibration problems of coupled shells, ranging from the classical thin shell theory [9,11,12], the shear deformation theory [7,13] to 3-D elasticity theory [10]. A large number of methods were also developed in this area, which include, but are not limited to, Ritz method, transfer matrix method, dynamic stiffness matrix method, pseudospectral method, variational method, state space method, discrete singular convolution method, finite element method, etc. When using these methods, it may not be possible to express the solution of the corresponding equations of motion as a closed form. Instead, the solution function is approximated by the higher-order function or Fourier series (harmonic function). That is to say, smooth and continuous basis functions are essential for numerically solving partial differential equations.

Recently, a new spectral collocation method based on integration scheme rather than conventional differentiation has been developed rapidly because of its simplicity, versatility, and various basis functions allowing the development of fast and accurate adaptive algorithms [23–26]. The advantages of this integration scheme lie in: (1) it does not require the basis function to be differentiable or continuous; (2) various boundary conditions can be easily handled; (3) no special integration technique such as Gauss integrals is needed; (4) the integration involved is a smooth operation and thus is more numerically stable. An important aspect for successful application of this integration scheme is how to choose the basis function. Chen and Hsiao [23] proposed an integral method based on the Haar wavelet series. Mai-Duy and Tanner [24] adopted the Chebyshev polynomials for numerically solving two-dimensional biharmonic boundary-value problems. Mai-Duy and Tran-Cong [25] used the thin plate splines for numerical solution of differential equations. Ngo-cong at al. [26] presented an integrated radial basis function method based on multi-quadric (MQ) for free vibration analysis of laminated composite plates. Then, these basis functions have been applied to a large number of problems and achieved very fruitfully [27–33]. The process of the method employed in this paper for the numerical solution is the representation of the highest derivatives appearing in the equations instead of the solution function itself by the truncated basis function. Then lower order derivatives and the function itself are obtained by integration. The constants appearing from the integrating process are determined by given boundary conditions. Also, the collocation algorithm can be viewed as a very effective meshless technique.

In this study, the spectral collocation method is employed for the modeling and vibration analysis of the coupled laminated shell structures with arbitrary elastic-support boundary conditions, where integrated orthogonal polynomials are applied with respect to the meridional direction and Fourier series are selected for the circumferential direction of each individual segment. The unknown constants arising from the integrating process are determined by the interface and boundary conditions of the coupled shells, and thus the equations of motion as well as the continuous and boundary condition equations are transformed into a set of algebraic equations. Then natural frequency of the coupled laminated shells is obtained by solving algebraic equations. Versatility and efficiency of the numerical method are tested through a series of free vibration analysis of coupled shell structures, and the results are compared with those existing solutions in other publications. In addition, effects of some geometrical and material parameters on the natural frequency of coupled laminated shells are discussed and some selected mode shapes are presented.

2. Theoretical formulations

2.1. The basis functions and their integrals

In order to demonstrate the utility and versatility of the proposed numerical technique, three sets of polynomials/series are applied to expand the highest derivatives of displacements and rotations of each shell segment in the meridional direction, they are respectively

(1) Chebyshev orthogonal polynomials of first kind (COPFK)

$$T_0(\xi)=1,\quad T_q(\xi)=\frac{q}{2}\sum_{\ell=0}^{[q/2]}(-1)^{\ell}\frac{2^{q-2\ell}(q-\ell-1)!}{\ell!(q-2\ell)!}\xi^{q-2\ell},\quad q>0. \tag{1.a}$$

(2) Chebyshev orthogonal polynomials of second kind (COPSK)

$$U_0(\xi)=1,\quad U_q(\xi)=\sum_{\ell=0}^{[q/2]}(-1)^{\ell}\frac{(q-\ell)!}{\ell!(q-2\ell)!}(2\xi)^{q-2\ell},\quad q>0 \tag{1.b}$$

(3) Legendre orthogonal polynomials of first kind (LOPFK)

$$P_0(\xi)=1,\quad P_q(\xi)=\sum_{\ell=0}^{[q/2]}(-1)^{\ell}\frac{(2q-2\ell)!}{2^q\ell!(q-\ell)!(q-2\ell)!}\xi^{q-2\ell},\quad q>0 \tag{1.c}$$

where $-1\le\xi\le1$ and $[q/2]$ is the integer part of $q/2$. The considered interval $[-1,1]$ is discretized by the non-uniform Chebyshev–Gauss–Lobatto points [30]; the coordinates of grid points are denoted by ξ_l.

$$\xi_l=-\cos\left(\frac{(l-1)\pi}{N-1}\right),\quad l=1,2,\ldots,N \tag{2}$$

where N is the total number of sampling points used to discretize the domain of interest. If we want to solve an nth order partial differential equation (PDE), the following integrals are required:

$$p_{\eta,q}(\xi)=\underbrace{\int_{-1}^{\xi}\int_{-1}^{\xi}\cdots\int_{-1}^{\xi}}_{\eta-times}h_q(\xi)d\xi^{\eta}\quad \eta=1,2,\ldots,n,\quad q=0,1,\ldots,N-1. \tag{3}$$

where h_q represents the above three sets of polynomials/series. The case $\eta=0$ corresponds to the orthogonal polynomial function itself. These integrals can be calculated analytically. In the case $q=0$, we have $p_{\eta,q}(\xi)=\xi^{\eta}/\eta!$; and in the case $q>0$ we obtain the integrals respectively as follows:

$$p_{\eta,q}(\xi)=\frac{q}{2}\sum_{\ell=0}^{[q/2]}(-1)^{\ell}\frac{2^{q-2\ell}(q-\ell-1)!}{\ell!(q-2\ell+\eta)!}\xi^{q-2\ell+\eta} \tag{4.a}$$

$$p_{\eta,q}(\xi)=\sum_{\ell=0}^{[q/2]}(-1)^{\ell}\frac{2^{q-2\ell}(q-\ell)!}{\ell!(q-2\ell+\eta)!}\xi^{q-2\ell+\eta} \tag{4.b}$$

$$p_{\eta,q}(\xi)=\sum_{\ell=0}^{[q/2]}(-1)^{\ell}\frac{(2q-2\ell)!}{2^q\ell!(q-\ell)!(q-2\ell+\eta)!}\xi^{q-2\ell+\eta} \tag{4.c}$$

For solving boundary value problems, the values $p_{\eta,q}(-1)$ and $p_{\eta,q}(1)$ should be calculated in order to satisfy boundary