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Exact solution for magneto-thermo-elastic behaviour of double-walled cylinder made of an inner FGM and an outer homogeneous layer

Abbas Loghman *, Hossein Parsa

Department of Solid Mechanics, Faculty of Mechanical Engineering, University of Kashan, Islamic Republic of Iran

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ABSTRACT

In this article, magneto-thermo-elastic response of a thick double-walled cylinder made from a functionally graded material (FGM) interlayer and a homogeneous outer layer is studied. The material property of the FGM layer varies along radius based on the power law distribution. Radial, circumferential and effective stresses as well as radial displacement under an applied internal pressure with and without the effect of magnetic field and thermal load are studied for five different material in-homogeneity parameters ($\beta = 0, \pm 1, \pm 2$). It has been shown that the material in-homogeneity parameter significantly affect the stress distribution in the FGM layer and consequently in the outer homogeneous layer as well. Therefore by selecting a suitable material parameter β one can control stress distribution in both FGM and homogeneous layers. It has been found that under thermo-magneto-mechanical loading minimum effective stress distribution and the minimum radial displacement can be achieved by selecting an appropriate material parameter ($\beta = -2$) in the FGM layer.

1. Introduction

A thick-walled single block homogeneous cylinder subjected to an internal pressure is suffering from highly tensile circumferential and compressive radial stresses at the inner surface imposing high shear stress inside the vessel [1]. For decades, shrink-fit and autofrettage of cylinders have been employed to overcome this deficiency. In both cases highly compressive circumferential stresses are created at the inner surface of the vessel leaving the outer surface in tension [2]. Design of autofrettage and compound cylinders are still used to withstand high internal pressures [3]. Material tailoring and analysis of functionally graded structures have also been an active area of research in the past decade [4]. In making structural components to withstand the combined thermal, mechanical and dynamical loadings multi-walled cylinders composed of FGM and homogeneous layers have attracted many researchers attention [5]. Mithchell and Reddy [6] have studied an embedded piezoelectric layer in composite cylinders. Transient plane-strain responses of multilayered elastic cylinders to axisymmetric impulse have been considered by Yin and Yue [7]. Stress wave propagation in laminated piezoelectric spherical shells under thermal shock and electric excitation has been investigated by Dai and Wang [8]. Dynamic response of a multilayered orthotropic piezoelectric hollow cylinder for axisymmetric plane strain condition has been presented by Wang and Chen [9]. Dynamic thermoelastic behaviour of a double-layered hollow cylinder with an FGM layer has been presented by Dai and Rao [10]. Also effect of magnetic field on stresses and displacements of axisymmetric structures has been considered by many researchers in recent years. Magnetothermoelastic interactions in hollow structures of functionally graded material subjected to mechanical loads have been considered by Dai and Fu [11]. An analytical method for magnetothermoelastic analysis of functionally graded hollow cylinders was also presented by Dai et al. [12]. The aim of this paper is to investigate the magnetothermoelastic response of double-walled cylinder made of an inner FGM and an outer homogeneous layer and finding the possible material property in the FGM layer for which minimum effective stress and displacement occur throughout the thickness of the vessel.

2. Geometry, material properties and loading condition

A thick double-walled hollow long cylinder with an inner radius a_1, outer radius a_3 and an interface radius of a_3 is considered. The interlayer is FGM the property of which varies along radius based on the power law distribution. The outer layer is made from homogeneous material. Loading is composed of internal pressure, external pressure, a uniform magnetic field in axial direction and a uniform temperature field. It is assumed that no separation will occur at the interface longitudinally or radially

* Corresponding author. Tel.: +98 361 5912425; fax: +98 361 5912424.
E-mail address: aloghman@kashanu.ac.ir (A. Loghman).

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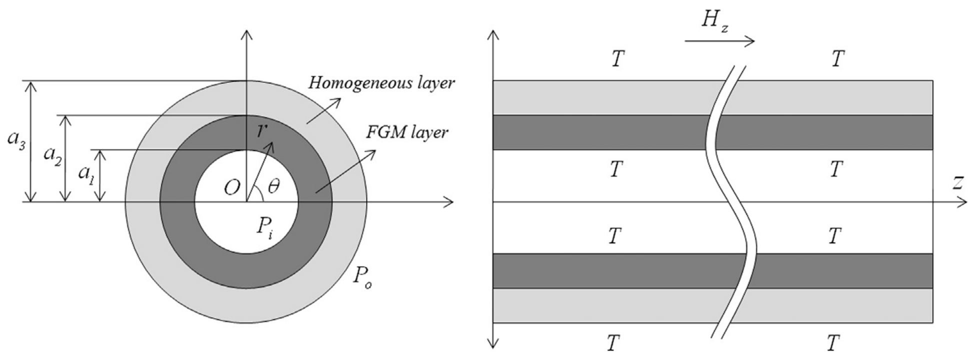

Fig. 1. Schematic of double-walled cylinder made of an inner FGM and an outer homogeneous layer.

for all loading combinations. Geometry of the problem is illustrated in Fig. 1.

3. Formulation of magneto-thermo-elastic analysis

The total strain is made up of elastic and thermal strains as

$$\varepsilon_{ij}=\varepsilon_{ij}^{E}+\alpha T\delta_{ij} \tag{1}$$

where the superscript E refer to the elastic strain, α is the coefficient of linear thermal expansion and δ_{ij} is the Kronecker delta. In this research cylindrical coordinate system is employed and axial symmetry is considered. The strain–displacement relationship for a long cylinder under axisymmetric loading condition is written as

$$\varepsilon_r=\frac{\partial u}{\partial r} \tag{2a}$$

$$\varepsilon_\theta=\frac{u}{r} \tag{2b}$$

Radial, circumferential and axial stresses under axisymmetric geometry and loading condition are principal stresses. They can be written in terms of total strains from Eq. (1) and then using strain–displacement relations and plane-strain condition yields

$$\begin{cases} \sigma_{rk}=c_{1k}(r)\frac{\partial u}{\partial r}+c_{2k}(r)\frac{u}{r}-\lambda_{1k}T(r) \\ \sigma_{\theta k}=c_{2k}(r)\frac{\partial u}{\partial r}+c_{1k}(r)\frac{u}{r}-\lambda_{1k}T(r) \\ \sigma_{zk}=\nu(\sigma_{rk}+\sigma_{\theta k})-E_k(r)\alpha_k(r)T(r) \end{cases} \tag{3}$$

where the subscript ($k=f$ and $k=h$) will be used for FGM layer and homogeneous layer respectively. The coefficients in the above equation are written as

$$c_{1k}(r)=\frac{E_k(r)(1-\nu_k)}{(1+\nu_k)(1-2\nu_k)},\quad c_{2k}(r)=\frac{E_k(r)\nu_k}{(1+\nu_k)(1-2\nu_k)}$$
$$\lambda_{1k}=\frac{E_k(r)\alpha_k(r)}{1-2\nu_k} \tag{4}$$

The equation of equilibrium for the cylinder in the presence of magnetic field in axial direction is written as follows:

$$\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}+f(r)=0 \tag{5}$$

where $f(r)$ is Lorentz's force which according to [13], [14] and [15] is defined as

$$f=q\vec{V}\times\vec{B}=\vec{J}\times\mu\vec{H}=\mu(\vec{J}\times\vec{H}) \tag{6}$$

where

$$\overleftarrow{J}=Curl\,\vec{h}\quad \vec{h}=Curl(\vec{U}\times\vec{H})$$
$$\vec{U}=(u,0,0)\quad \vec{H}=(0,0,H_z)$$
$$\vec{U}\times\vec{H}=H_z(0,-u,0) \tag{7}$$

thus

$$\begin{aligned}\vec{h}&=curl(\vec{U}\times\vec{H})=\nabla\times(\vec{U}\times\vec{H})=\nabla\\ &\quad\times(H_z(0,-u,0))=H_z[\nabla\times(0,-u,0)]\\ &=H_z\left[e_r\left(\frac{1}{r}\frac{\partial(0)}{\partial\theta}-\frac{\partial(-u)}{\partial z}\right)+e_\theta\left(\frac{\partial(0)}{\partial z}-\frac{\partial(0)}{\partial r}\right)\right.\\ &\quad\left.+e_z\left(\frac{1}{r}\frac{\partial}{\partial r}(r(-u))-\frac{1}{r}\frac{\partial(0)}{\partial\theta}\right)\right]=H_z\left[e_r(0)+e_\theta(0)+e_z\left(-\frac{\partial u}{\partial r}-\frac{u}{r}\right)\right]\end{aligned} \tag{8}$$

and

$$\begin{aligned}\vec{J}&=curl\,\vec{h}=\nabla\times\vec{h}=H_z\left[\nabla\times\left(0,0,-\frac{\partial u}{\partial r}-\frac{u}{r}\right)\right]\\ &=H_z\left[e_r(0)+e_\theta\left(-\frac{\partial}{\partial r}\left(-\frac{\partial u}{\partial r}-\frac{u}{r}\right)\right)+e_z(0)\right]=H_z\left(0,\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right),0\right)\end{aligned} \tag{9}$$

and

$$\vec{J}\times\vec{H}=H_z\left(0,\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right),0\right)\times(0,0,H_z)=H_z^2\left(\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right),0,0\right) \tag{10}$$

Substituting Eq. (10) into Eq. (6) Lorentz's force is obtained as

$$f(r)=\mu H_z^2\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right) \tag{11}$$

where u is the radial displacement.

In the FGM layer the material properties are assumed to be power functions of radius as

$$E_f=E_{0f}r^\beta,\quad \alpha_f=\alpha_{0f}r^\beta,\quad \mu_f=\mu_{0f}r^\beta \tag{12}$$

Substituting Eqs. (3) and (11) into equilibrium Eq. (5) the following differential equation is obtained:

$$D_1\frac{\partial^2 u}{\partial r^2}+D_2\frac{\partial u}{\partial r}+D_3u+D_4=0 \tag{13}$$

In which coefficients of the above differential equation for the FGM and homogeneous layers are written as

$$D_{1f}=D'_{1f}r^\beta$$
$$D_{2f}=D'_{2f}r^{\beta-1}$$
$$D_{3f}=D'_{3f}r^{\beta-2}$$
$$D_{4f}=D'_{4f}r^{2\beta-1}-\frac{E_{0j}\alpha_{0j}r^{2\beta}}{1-2\nu_j}\frac{\partial T(r)}{\partial r} \tag{14a}$$

And

$$D_{1h}=D'_{1h}$$
$$D_{2h}=D'_{2h}r^{-1}$$
$$D_{3h}=D'_{3h}r^{-2}$$
$$D_{4h}=-\frac{E_h}{1-2\nu_h}\frac{\partial T(r)}{\partial r} \tag{14b}$$