International Journal of Mechanical Sciences 88 (2014) 122–137

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Free and forced vibration analysis of coupled conical–cylindrical shells with arbitrary boundary conditions

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ARTICLE INFO

Article history:
Received 12 February 2014
Received in revised form
2 June 2014
Accepted 3 August 2014
Available online 9 August 2014

Keywords:
Vibration analysis
Coupled conical–cylindrical shells
Modified Fourier series
Arbitrary boundary conditions

ABSTRACT

This paper presents a free and forced vibration analysis of coupled conical–cylindrical shells with arbitrary boundary conditions using a modified Fourier–Ritz method. Under the current framework, regardless of the boundary conditions, each of the displacement components of both the conical and cylindrical shells are expanded invariantly as a modified Fourier series, which is composed of a standard Fourier series and closed-form supplementary functions introduced to accelerate the convergence of the series expansion and remove all the relevant discontinuities at the boundaries and the junction between the two shell components. All the expansion coefficients are determined by using the Rayleigh–Ritz method as the generalized coordinates. By using the present method, a unified solution for the coupled conical–cylindrical shells with classical and non-classical boundary conditions can be directly derived without the need of changing either the equations of motion or the expressions of the displacements. The reliability and accuracy of the present method are validated by comparison with FEM results and those from the literature. Studies on the effects of dimensional and elastic restraint parameters on the free vibrations are also reported. Investigation on vibration of the conical–cylindrical–conical shell combination shows the extensive applicability of present method for more complex shell combinations. New numerical examples are also conducted to illustrate the forced vibration behavior of the coupled conical–cylindrical shell subjected to the excitation forces in different directions.

1. Introduction

The coupled conical–cylindrical shell is a shell combination of great interest in practical engineering applications, such as torpedo, rocket, tower and naval hulls of submarines, owing to its excellent mechanical and physical properties. In practical designs, the shell combination is commonly used as the foundation structure subjected to the intricate environment and dynamic loads, resulting in vibration, fatigue damage and radiated noise. Since the 50s of last century, much effort has been done to study the vibration characteristics of shell structures. Many researchers, such as Donnell, Mushtari, Flügge, to name a few, have developed various shell theories based on different simplifying assumptions and approximations. These works have been well summarized by Leissa [1], Markuš [2] and Qatu [3,4]. Recently, some new approaches have been adopted to analyze the vibration behaviors of the shell of revolutions. Wu and Lee [5] used the method of differential quadrature for free vibration analysis of laminated conical shells with variable stiffness. The equations of motion and the boundary conditions in the whole domain are replaced by a system of simultaneously algebraic equations with respect to the function values of all the sampling points. It should be noted that most of the literature concentrates on the elementary shell configurations, such as circular cylindrical, conical and spherical shells rather than the shell combinations. Compared to the elementary shell structures, different components of the shell combination find their natural description in different physical co-ordinate systems and a problem will be caused by the matching of the interface continuity conditions between the substructures, which leads directly to the difficulty of obtaining the accurate vibration solution. The finite element method (FEM) computer programs such as ANSYS, ABAQUS and NASTRAN have been well developed and applied for vibration analysis of these complex shell combinations. However, there are two main disadvantages in the computation procedure: firstly, a great number of interior points are needed if one wants to obtain the accurate solution at high frequencies, which would finally increase the computation time and storage requirement; secondly, it is hard to identify the mode shapes corresponding to the certain natural frequencies in the modal analysis. Thus, developing an accurate and efficient

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http://dx.doi.org/10.1016/j.ijmecsci.2014.08.002
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method is of considerable technical significance to characterize the vibration of the coupled conical–cylindrical shell combinations.

In the last few decades, a few but not many publications focused on the vibration analysis of the coupled conical–cylindrical shell have been reported in the literature. Kalnins [6] and Rose et al. [7] used the classic bending theory to examine rotationally symmetric shells. Hu and Raney [8] proposed a multi-segmental numerical integration technique to obtain the analytical results for joined conical–cylindrical shells. The interface continuity conditions are imposed on the segments of the shell combination in their study and good agreement is observed compared with experimental results. The transfer matrix method was used by Irie et al. [9] to investigate the vibration behavior of a conical–cylindrical shell combination. Efraim and Eisenberger [10] studied the free vibration behavior of segmented axisymmetric shells by using a power series solution and obtained relatively accurate natural frequencies. Galletly and Mistry [11] obtained the natural frequencies of cylindrical shells clamped at one end and closed at the other by different types of shells, including cones, hemispheres, ellipsoids, etc. by using variational finite differences and finite elements. Their study indicates the fact that the in-plane boundary conditions have considerable influence on the natural frequencies. A local-global B-spline finite element method was presented by Benjeddou [12] for modal analysis of the coupled shells of revolution. Caresta and Kessissoglou [13] presented a classical approach to investigate the vibration characteristics of isotropic coupled conical–cylindrical shells. In their study, a wave solution is adopted to solve the cylindrical shell equations while the conical shell equations are solved by using a power series solution. These two solutions are coupled together by the means of the continuity conditions at the junction. A three-dimensional finite element method is used by EI Damatty et al. [14] to study numerically the dynamic behavior of a joined conical–cylindrical shell. Qu et al. [15,16] proposed a modified variational approach to analyze the free and forced vibrations of ring-stiffened conical, cylindrical and spherical shell combinations. The coupled shell structure is partitioned into appropriate shell segments and all essential continuity constraints on segment interfaces are imposed by means of a modified variational principle and least-squares weighted residual method. The dynamical responses of the shell combination obtained by the method agree well with those from FEM program. Experiments and numerical simulations are adopted to investigate the plastic energy absorption behavior of expansion tubes under axial compression by conical–cylindrical die was investigated by Yang et al. [17]. Free vibration analysis of coupled cross-ply laminated conical shells was presented by Kouchakzadeh and Shakouri [18]. In their study, the cross-ply conical shell combination is considered as the general case of cylindrical–conical shells, joined cylinder-plates and cone-plates. The continuity conditions at the joining section of the cones ware achieved by the extraction of the appropriate expressions among stress resultants and deformations.

Mathematically, compared to directly solve the actual problems, it is easier to obtain the solution for the shell combinations by describing the boundary value and eigenvalue problems in a variational form. This is due to the fact that expanding the solution over a set of suited admissible functions can achieve the extreme or stationary value of some kind of energy functional for a coupled conical–cylindrical shell. As the classical variational approach, the Rayleigh–Ritz method has found its efficiency in the vibration analysis of the shell combinations. Monterrubio [19] presented the Rayleigh–Ritz method and the penalty function method to solve the vibration problem of shallow shells of rectangular planform with spherical, cylindrical and hyperbolic paraboloidal geometries with classical boundary conditions. Lee et al. [20] used the Rayleigh–Ritz method to investigate the free vibration of a joined hemispherical-cylindrical shell. At the joint part of the shell combination in the study, the hemispherical-cylindrical shell is assumed to have a free boundary condition while the cylindrical shell has a simply supported boundary constraints.

From the review of the literature, most of previous works on the vibration analysis of the coupled conical–cylindrical shells just concentrate on the cases with classical boundary conditions rather than the general elastic boundary conditions. Even so, either theoretical formulations or the admissible functions of the displacements have to be changed if one wants to obtain the solution for the cases with different boundary restraints. Furthermore, the general boundary conditions are often encountered in practical engineering applications compared to the classical boundary conditions since the support types of practical structures are always complicated and variable in nature. Li [21] proposed originally the modified Fourier series solution for the vibration analysis of beams with general elastic restraints. The flexural displacements are expressed by an improved Fourier series, which is composed a standard Fourier series and an auxiliary polynomial function introduced to remove all the relevant discontinuities at the boundaries. Subsequently, this method was extensively adopted for the vibration analysis of rectangular plates, circular cylindrical, conical, etc. shells with classical and non-classical boundary conditions [22–27]. Ma et al. [28] investigated the active control of an elastic cylindrical shell coupled to a vibration isolation system. The cylindrical shell is simply supported at its two ends and four active control strategies are evaluated in terms of the acoustic power radiated from the supporting shell. In practice engineering applications, the foundation structures are always formulated by shell combinations with non-classical boundary conditions rather than the simply supported elementary shells. To the author's best knowledge, few publications focused on the vibration analysis of the coupled conical–cylindrical shell with general elastic boundary conditions have been reported.

The main objective of this paper is to develop an alternative and unified solution for the vibration analysis of the coupled conical–cylindrical shell with general elastic boundary conditions. The Reissner's thin shell theory is used to formulate the theoretical models of the conical and cylindrical shell components. Regardless of the boundary conditions, each displacement of the two shell components is invariantly expressed by the modified Fourier series composed of a standard Fourier series and closed-form auxiliary functions. The introduction of the auxiliary functions can not only remove all the potential discontinuities at the boundaries and the junction between the two shells, but also ensure and accelerate the convergence of the series expansions. All the expansion coefficients are determined by using the Rayleigh–Ritz procedure as the generalized coordinates. The accuracy and convergence of present method are validated by comparison with FEM results and those from the literature. The effects of semi-vertex angle of the cone and the elastic restraint parameters on the free vibration behavior of the shell combination are studied. New examples are also conducted to analyze the forced vibration responses of the coupled conical–cylindrical shell subjected to the driving forces in different directions.

2. Theoretical formulation

2.1. System description

The geometry and co-ordinate systems for the coupled conical–cylindrical shell are depicted in Fig. 1. The conical shell is described with the (x_c, θ_c, r_c)coordinate system, in which x_c is measured along the generator of the cone starting at its small edge, θ_c is the circumferential co-ordinate and r_c is perpendicular to middle surface of the conical shell. The displacements of the conical shell with respect to this coordinate system are described by u_c,v_c and