International Journal of Mechanical Sciences 78 (2014) 1–7

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

A multi-scaling approach to predict hydraulic damage of poromaterials

A. Karrech [a,*], C. Schrank [d,b], R. Freij-Ayoub [c], K. Regenauer-Lieb [b,c]

[a] School of Civil and Resource Engineering, University of Western Australia, 35 Stirling Hwy, Crawley WA 6009, Australia
[b] School of Earth and Environment , University of Western Australia, Crawley, Australia
[c] CSIRO Earth Science and Resource Engineering, Kensington, Australia
[d] School of Earth, Environmental and Biological Sciences, Queensland University of Technology, Brisbane, Australia

ARTICLE INFO

Article history:
Received 8 March 2013
Received in revised form
18 September 2013
Accepted 8 October 2013
Available online 17 October 2013

Keywords:
Hydraulic damage
Coupled systems
Hydro-mechanics
Geomaterials

ABSTRACT

The purpose of this paper is to introduce the concept of hydraulic damage and its numerical integration. Unlike the common phenomenological continuum damage mechanics approaches, the procedure introduced in this paper relies on mature concepts of homogenization, linear fracture mechanics, and thermodynamics. The model is applied to the problem of fault reactivation within resource reservoirs. The results show that propagation of weaknesses is highly driven by the contrasts of properties in porous media. In particular, it is affected by the fracture toughness of host rocks. Hydraulic damage is diffused when it takes place within extended geological units and localized at interfaces and faults.

1. Introduction

Multi-scaling is the branch of science which describes homogenization and data assimilation. Homogenization predicts the overall behavior of materials based on local considerations. Data assimilation includes the opposite process which deduces the description of local processes from the overall behavior of materials. Multi-scaling to predict the constitutive behavior of geomaterials has been the subject of intensive research. This problem does not necessarily refer to the transition from a particular unit of length to another as often argued but it allows to predict the behavior of geomaterials on the macro-scale (scale of homogeneity) from local micro-scale (scale of heterogeneities) and vice versa. In the context of geomechanics, the micro-scale can capture grains, agglomerates, rocks of different dimensions, and/or entire geological layers. The objective of multi-scaling is to replace the behavior of locally complex bodies by a fictitious homogeneous material, which exhibits equivalent properties as the complex bodies on the global scale (upscaling), or to derive locally complex properties from a globally homogeneous material (downscaling).

In this spirit, we present a continuum hydraulic damage approach based on simple concepts of linear fracture mechanics and coherent classical homogenization techniques. We also introduce a numerical implementation of this approach using the finite element method. At a second stage we apply the developed framework to the problem of fault reactivation within resource reservoirs. The case study proposed herein is hypothetical. Yet, it reflects a realistic scenario which could be encountered in geothermal energy harnessing, carbon dioxide geo-sequestration, hydraulic stimulation of shale gas reservoirs, and/or oil and gas recovery.

2. Micro–macro scale description

2.1. Basic principles of homogenization

The purpose of this subsection is to provide a concise overview on the basic principles of homogenization. A detailed survey can be found in Zaoui [17]. The theory of homogenization predicts the effective behavior of composite materials knowing their local configurations, texture and morphology. The outcome of this theory is valid as long as a representative volume element (RVE) plays the same mechanical role as the equivalent homogeneous medium (EHM). This means that the stress and strain fields $\boldsymbol{\Sigma}$ and $\boldsymbol{E}$, solutions of the boundary value problem on the EHM, must coincide with the space average of $\boldsymbol{\sigma}(\boldsymbol{x})$ and $\boldsymbol{\epsilon}(\boldsymbol{x})$ over any RVE centered on $\boldsymbol{x}$. The tensors $\boldsymbol{\sigma}(\boldsymbol{x})$ and $\boldsymbol{\epsilon}(\boldsymbol{x})$ are, respectively, the stress and strain that would have been obtained if the boundary value problem was solved on the domain by taking into account all the local morphologic complexities. The size of an RVE should be (i) small enough compared to the structure to be studied and (ii) big enough compared to the heterogeneities. This view is

* Corresponding author. Tel.: +61 84883523.
E-mail address: ali.karrech@uwa.edu.au (A. Karrech).

0020-7403/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijmecsci.2013.10.010

expressed quantitatively by the Hill–Mandel theorem as discussed in the next subsections.

The homogenization theory relies considerably on the conditions of homogeneous boundary conditions [8,9,17]. The condition of *homogeneous stress boundary condition* stipulates that the surface traction prescribed on the boundaries is given by $\boldsymbol{T}^g = \boldsymbol{\Sigma} \cdot \boldsymbol{n}$ where $\boldsymbol{n}$ is an outward normal unit vector. This condition shows that the space average $\overline{\sigma}$ of stress in the RVE coincides with $\boldsymbol{\Sigma}$. Similarly, the condition of *homogeneous strain boundary condition* expressed as $\boldsymbol{u}^g = \boldsymbol{E} \cdot \boldsymbol{x}$ shows that the space average $\overline{\epsilon} = \boldsymbol{E}$. Note that this condition is acceptable as long as the heterogeneities are small as compared to the RVE. Therefore, from the homogeneous boundary conditions it can be shown that

$$\overline{\epsilon^*} = \frac{1}{|\Omega|}\int_\Omega \epsilon^*(\boldsymbol{x})\, d\Omega = \boldsymbol{E} \quad \text{and} \quad \overline{\sigma^*} = \frac{1}{|\Omega|}\int_\Omega \sigma^*(\boldsymbol{x})\, d\Omega = \boldsymbol{\Sigma} \tag{1}$$

where $|\Omega|$ is the volume of the RVE. The superscript (*) in the above equation means that the stress and strain fields, which are solutions of the boundary value problem, are not the only ones that satisfy the equalities (1). Any stress and strain fields, that satisfy stress equilibrium and compatibility with the boundary conditions, are admissible. Homogeneous boundary conditions also result in the Hill lemma which reads

$$\overline{\sigma^*(\boldsymbol{x}) : \epsilon^*(\boldsymbol{x})} = \overline{\sigma^*}(\boldsymbol{x}) : \overline{\epsilon^*(\boldsymbol{x})} = \boldsymbol{\Sigma} : \boldsymbol{E} \tag{2}$$

The homogenization procedure requires either the deformation or the stress to obey homogeneous boundary conditions. Therefore, it is possible to obtain one of the following relationships:

$$\epsilon^*(\boldsymbol{x}) = \mathcal{A}(\boldsymbol{E}) \quad \text{or} \quad \sigma^*(\boldsymbol{x}) = \mathcal{B}(\boldsymbol{\Sigma}) \tag{3}$$

which express the local deformation as a function of the overall strain or the local stress as a function of the overall stress. In particular, if the behavior is assumed to be linear, Eq. (3) can be rewritten in the following form:

$$\epsilon^*(\boldsymbol{x}) = \mathbb{A}(\boldsymbol{x}) : \boldsymbol{E} \quad \text{or} \quad \sigma^*(\boldsymbol{x}) = \mathbb{B}(\boldsymbol{x}) : \boldsymbol{\Sigma} \tag{4}$$

The fourth order tensors $\mathbb{A}$ and $\mathbb{B}$ are known as the localization (or concentration) tensors. Using Eqs. (1), it can be shown that they verify $\overline{\mathbb{A}(\boldsymbol{x})} = \overline{\mathbb{B}(\boldsymbol{x})} = \mathbb{I}$, where $\mathbb{I}$ is the identity tensor. Alternating the homogeneous boundary conditions which result in (4) and applying Hooke's law locally shows that

$$\boldsymbol{\Sigma} = \overline{\mathbb{S} : \mathbb{A} : \boldsymbol{E}} = \mathbb{S}_{hom} : \boldsymbol{E} \quad \text{or} \quad \boldsymbol{E} = \overline{\mathbb{C} : \mathbb{B} : \boldsymbol{\Sigma}} = \mathbb{C}_{hom} : \boldsymbol{\Sigma} \tag{5}$$

where $\mathbb{S}$ and $\mathbb{C}$ are the stiffness and compliance tensors, respectively. The subscript "hom" refers to the homogeneous or equivalent property. The condition of equivalence of the homogeneous boundary conditions is expressed by the so called Hill–Mandel theorem:

$$\mathbb{S}_{hom} : \mathbb{C}_{hom} = \mathbb{I} + \mathcal{O}\left(\frac{d_{inc}}{d_{rve}}\right)^3 \tag{6}$$

which states that the equivalence is ensured when the characteristic dimension of the inclusions d_{inc} is small as compared to the characteristic dimension of the RVE, d_{rve}. The equivalence is ensured with an error of the third order with respect to the above-mentioned ratio. For instance, if the representative volume element is taken 3 times bigger than the dimension of the heterogeneities, the overall elasticity properties can be estimated with an error of the order 1%. This is important especially for geological applications where continuum mechanics approaches are used to describe complex local textures.

2.2. Effective properties of fractured media

In the last subsection, we recalled the basic principles of homogenization including the homogeneous boundary conditions and how they can be used to derive Hill's lemma. We also explained the concept of scale separation which is well described by Hill–Mandel's theorem. In this subsection, we use the homogenization theory to describe the effective behavior of materials embedding randomly distributed fractures. This description summarizes the results of Eshelby [6], Mori and Tanaka [16] and Benveniste [1], which are useful to develop our numerical procedure.

Consider an elliptical inclusion within an infinite elastic domain. A deformation $\boldsymbol{E}_a$ is imposed at the infinite boundaries of the domain. According to Eshelby [6], the deformation within the inclusion can be expressed as

$$\epsilon_i(\boldsymbol{x}) = (\mathbb{I} + \mathbb{P}_i(\boldsymbol{x}) : (\mathbb{S}_i - \mathbb{S}_s))^{-1} : \boldsymbol{E}_a = (\mathbb{I} - \mathbb{E}_i)^{-1} : \boldsymbol{E}_a \tag{7}$$

where $\mathbb{P}_i$ is Hill's tensor, $\mathbb{S}_i$ is the stiffness tensor of the inclusion, and $\mathbb{S}_s$ is the stiffness tensor of the solid matrix. Since the inclusion is an empty pore space, $\mathbb{S}_i$ is identically zero and $\mathbb{E}_i = \mathbb{P}_i : \mathbb{S}_s$ Eshelby's tensor. Now consider multiple pores in an infinite elastic space. Their interaction can be taken into account by changing the boundary conditions at infinity. Within a RVE, the micro–macro strain compatibility delivers the overall deformation:

$$\boldsymbol{E} = \overline{\epsilon(\boldsymbol{x})} = \varphi_s \epsilon_s + \sum_i \varphi_i \epsilon_i \tag{8}$$

where φ_s is the volume fraction of the solid phase and φ_i is the volume fraction of the ith inclusion. Substituting for $\epsilon_s = \boldsymbol{E}_a$ and ϵ_i from Eq. (7) into Eq. (8) results in

$$\boldsymbol{E}_a = \left(\varphi_s \mathbb{I} + \sum_i \varphi_i (\mathbb{I} - \mathbb{E}_i)^{-1}\right)^{-1} : \boldsymbol{E} \tag{9}$$

The stress can be expressed as $\sigma(\boldsymbol{x}) = \mathbb{S}_s : \epsilon_s(\boldsymbol{x})$ if $\boldsymbol{x}$ is within the solid phase and $\sigma(\boldsymbol{x}) = \mathbb{S}_i : \epsilon_i(\boldsymbol{x})$ if not. Therefore, averaging over the entire domain results in

$$\boldsymbol{\Sigma} = \frac{1}{|\Omega|}\int_\Omega \mathbf{1}_s \mathbb{S}_s : \epsilon_s(\boldsymbol{x})\, d\Omega + \sum_i \int_\Omega \mathbf{1}_i \mathbb{S}_i : \epsilon_i(\boldsymbol{x})\, d\Omega = \varphi_s \mathbb{S}_s : \boldsymbol{E}_a \tag{10}$$

where $\mathbf{1}_s$ and $\mathbf{1}_i$ are characteristic functions of the different phases. Therefore, it can be deduced that

$$\boldsymbol{\Sigma} = \mathbb{S}_{hom} : \boldsymbol{E} = \varphi_s \mathbb{S}_s : \overline{\mathbb{A}_s} = \varphi_s \mathbb{S}_s : \left(\varphi_s \mathbb{I} + \sum_i \varphi_i (\mathbb{I} - \mathbb{E}_i)^{-1}\right)^{-1} : \boldsymbol{E} \tag{11}$$

The above result is known as the Mori–Tanaka self-consistent homogenization scheme [16,1].

2.3. Effective poroelastic properties

To obtain effective poroelastic properties, we consider that the RVE is subjected to an overall deformation $\boldsymbol{E}$ and to an overall pore pressure P which homogenizes the fluid pressure within the porous medium. As suggested by Chateau and Dormieux [3], the deformation can be expressed as follows:

$$\epsilon(\boldsymbol{x}) = \mathbb{A}(\boldsymbol{x}) : \boldsymbol{E} - \boldsymbol{A}(\boldsymbol{x}) P \tag{12}$$

where $\boldsymbol{A}$ is a localization tensor. The stress tensor within the solid phase can be expressed as follows:

$$\sigma(\boldsymbol{x}) = \mathbb{S}_s : \mathbb{A}(\boldsymbol{x}) : \boldsymbol{E} - \mathbb{S}_s : \boldsymbol{A}(\boldsymbol{x}) P \tag{13}$$

The macroscopic stress can be obtained by averaging as follows:

$$\boldsymbol{\Sigma} = \frac{1}{|\Omega|}\left(\int_{\Omega_s} \sigma(\boldsymbol{x})\, d\Omega + \int_{\Omega_f} -P\mathbf{1}\, d\Omega\right) = \varphi_s \sigma_s - P\varphi \mathbf{1} \tag{14}$$

where $\varphi = (1 - \varphi_s)$. Substituting the expression of stress (13) into Eq. (14) shows that

$$\boldsymbol{\Sigma} = \mathbb{S}_{hom} : \boldsymbol{E} - \boldsymbol{B} P \tag{15}$$