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Zeroth-order shear deformation theory for functionally graded plates resting on elastic foundation

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ABSTRACT

This paper presents a zeroth-order shear deformation theory for bending and vibration analyses of functionally graded plates resting on elastic foundation. In the present theory, the shear deformation effect is incorporated in the in-plane displacements through the use of shear forces instead of rotational displacements as in existing shear deformation theories. Equations of motion and boundary conditions are derived from Hamilton's principle. Analytical solutions of simply supported plates are presented, and the obtained results are compared with available solutions to verify the accuracy of the present theory. Numerical results show that the present theory gives a very good prediction of bending and vibration responses of functionally graded plates resting on elastic foundation.

1. Introduction

Functionally graded materials (FGMs) are a class of composites that have continuous variation of material properties from one surface to another and thus eliminate the stress concentration found in laminated composites. A typical FGM is made from a mixture of two material phases, for example, a ceramic and a metal. The reason for the increasing use of FGMs in a variety of aerospace, automotive, civil, and mechanical engineering structures is that their material properties can be tailored to different applications and working environments [1]. To describe the interaction between the plate and foundation, several foundation models have been proposed. The simplest one is the Winkler or one-parameter model [2] which models the foundation as a series of separated springs without coupling effects between each other. This model was improved by Pasternak [3] by adding a shear spring to simulate the interactions between the separated springs in the Winkler model. The Pasternak or two-parameter model is widely used to describe the mechanical behavior of structure–foundation interactions and will be used here to simulate the interactions between the plate and foundation.

Since the shear deformation effect is more pronounced in FGMs, shear deformation theories should be used to analyze functionally graded (FG) plates. The first-order shear deformation theory (FSDT) developed by Mindlin [4] and Reissner [5] accounts for the transverse shear deformation effect, but it violates the traction-free boundary conditions on the top and bottom surfaces. A shear correction factor is therefore required to compensate for the difference between actual stress state and assumed constant stress state [6,7]. To avoid the use of the shear correction factor and obtain a better prediction of the transverse shear deformation and normal strains in FG plates, higher-order shear deformation plate theories (HSDTs) have been proposed. In general, HSDTs can be developed based on higher-order variations of the in-plane displacements [1,8–18] or both in-plane and transverse displacements [19–31] (i.e. quasi-3D theories). However, HSDTs are highly computational cost due to involving in many unknowns (e.g., theories by Neves et al. [27–29] with nine unknowns, Reddy [23] with eleven unknowns, Jha et al. [31] with twelve unknowns, Talha and Singh [21] and Natarajan and Manickam [24] with thirteen unknowns). Thus, Shimpi [32] proposed a zeroth-order shear deformation theory (ZSDT) which is simple to use.

The ZSDT accounts for the transverse shear deformation effect through the use of shear forces instead of rotational displacements as in existing shear deformation theories. The ZSDT contains the same five unknowns as in the FSDT, but satisfies the traction-free boundary conditions on the top and bottom surfaces of the plate without requiring any shear correction factor. The ZSDT was first developed by Shimpi [32] for isotropic plates and later extended by Ray [33] for laminated composite plates, which predicts accurate results for both isotropic and laminated composite plates. Therefore, it seems to be important to extend this theory to FG plates. In this paper, the ZSDT is extended and evaluated for FG plates resting on elastic foundation. Material properties of FG plates are assumed to vary according to a power law distribution of the volume fraction of the constituents. Pasternak model is used

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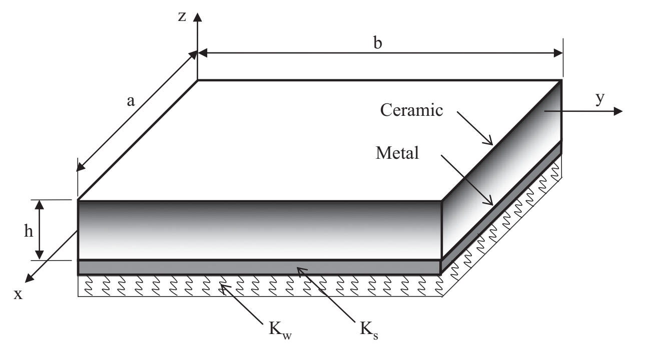

Fig. 1. Geometry and coordinate of rectangular plate resting on elastic foundation.

simulate the interactions between the plate and elastic foundation. Equations of motion and boundary conditions are derived from Hamilton's principle. Closed-form solutions of simply supported plates are presented. The obtained results are compared with the existing solutions to verify the accuracy of present theory in predicting the bending and vibration responses of FG plates.

2. Theoretical formulations

2.1. Kinematics

The displacement field of the ZSDT is chosen based on the assumption that the transverse shear stresses vary parabolically across the plate thickness and vanish on the plate surfaces, and consequently, there is no need to use shear correction factor. Based on this assumption, the following displacement field can be obtained [32,33]

$$u_1(x,y,z,t)=u(x,y,t)-z\frac{\partial w}{\partial x}+\frac{1}{\lambda_x}\left[\frac{3}{2}\left(\frac{z}{h}\right)-2\left(\frac{z}{h}\right)^3\right]Q_x(x,y,t)$$
$$u_2(x,y,z,t)=v(x,y,t)-z\frac{\partial w}{\partial y}+\frac{1}{\lambda_y}\left[\frac{3}{2}\left(\frac{z}{h}\right)-2\left(\frac{z}{h}\right)^3\right]Q_y(x,y,t)$$
$$u_3(x,y,z,t)=w(x,y,t) \tag{1}$$

where u, v, and w are the displacements of a point on the reference plane in the x, y, and z directions, respectively; h is the plate thickness; Q_x and Q_y are the transverse shear forces; and λ_x and λ_y are unknown constants determined based on the definition of the transverse shear forces as

$$Q_i=\int_{-h/2}^{h/2}\sigma_{iz}\,dz, \quad (i=x,y) \tag{2}$$

The nonzero linear strains associated with the displacement field in Eq. (1) are:

$$\varepsilon_x=\frac{\partial u}{\partial x}-z\frac{\partial^2 w}{\partial x^2}+f\frac{\partial Q_x}{\lambda_x\partial x} \tag{3a}$$

$$\varepsilon_y=\frac{\partial v}{\partial y}-z\frac{\partial^2 w}{\partial y^2}+f\frac{\partial Q_y}{\lambda_y\,\partial y} \tag{3b}$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2z\frac{\partial^2 w}{\partial x\,\partial y}+f\left(\frac{\partial Q_x}{\lambda_x\,\partial y}+\frac{\partial Q_y}{\lambda_y\,\partial x}\right) \tag{3c}$$

$$\gamma_{xz}=g\frac{Q_x}{\lambda_x} \tag{3d}$$

$$\gamma_{yz}=g\frac{Q_y}{\lambda_y} \tag{3e}$$

where $f=(3/2)(z/h)-2(z/h)^3$ and $g=df/dz=(3/2h)(1-4(z^2/h^2))$.

It is observed from Eqs. (3d) and (3e) that the transverse shear strains $(\gamma_{xz},\gamma_{yz})$ vary parabolically across the plate thickness and vanish at the plate surfaces $(z=\pm h/2)$, thus satisfying the traction free conditions for transverse shear stresses $(\sigma_{xz},\sigma_{yz})$. In this regard, it may be mentioned here that the theories proposed by Reddy [1] and Thai and Choi [12] also account for parabolic distribution of the transverse shear strains through the plate thickness. However, these theories use rotational displacements to account for the shear deformation effect whereas the ZSDT uses the shear forces to account for the same (see Eqs. (3d) and (3e)). It is worth noting that the transverse shear strain expressions in Eqs. (3d) and (3e) do not explicitly contain the rotational displacements due to the shear deformation effect. Hence, the present theory may be called as a zeroth-order shear deformation theory [33].

2.2. Constitutive equations

Consider FG plates made from a mixture of two material phases, for example, a metal and a ceramic as shown in Fig. 1. The properties of FG plates such as Young's modulus E and mass density ρ are assumed to vary through the plate thickness with a power law distribution of the volume fraction of the two materials as

$$E(z)=E_m+(E_c-E_m)\left(\frac{1}{2}+\frac{z}{h}\right)^p$$
$$\rho(z)=\rho_m+(\rho_c-\rho_m)\left(\frac{1}{2}+\frac{z}{h}\right)^p \tag{4}$$

where the subscripts m and c represent the metallic and ceramic constituents, respectively; and p is the power law index. The value of p equal to zero represents a fully ceramic plate, whereas infinite p indicates a fully metallic plate. Since the effects of the variation of Poisson's ratio ν on the response of FG plates are very small [34,35], it is assumed to be constant for convenience. The linear constitutive relations of a FG plate can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{Bmatrix}=\frac{E(z)}{1-\nu^2}\begin{bmatrix}1&\nu&0&0&0\\ \nu&1&0&0&0\\ 0&0&\frac{(1-\nu)}{2}&0&0\\ 0&0&0&\frac{(1-\nu)}{2}&0\\ 0&0&0&0&\frac{(1-\nu)}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \tag{5}$$

By substituting the expressions of the transverse shear strains γ_{xz} and γ_{yz} into Eq. (5) and the subsequent results of transverse shear stresses σ_{xz} and σ_{yz} into Eq. (2), unknown constants λ_x and λ_y are obtained as

$$\lambda_x=\lambda_y=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}g\,dz=\frac{1}{2(1+\nu)}\left[E_m+\frac{6(E_c-E_m)}{(p+2)(p+3)}\right] \tag{6}$$

2.3. Equations of motion

Hamilton's principle is used herein to derive the equations of motion of the ZSDT for FG plates resting on elastic foundation. The principle can be stated in analytical form as

$$0=\int_0^T(\delta U_P+\delta U_F+\delta V-\delta K)\,dt \tag{7}$$

where δU_P and δU_F are the variations of strain energy of the plate and foundation, respectively; δV is the variation of work done; and δK is the variation of kinetic energy. The variation of strain energy