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Static response of advanced composite plates by a new non-polynomial higher-order shear deformation theory

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ABSTRACT

The static responses of functionally graded plates are investigated by using an accurate recently developed higher order shear deformation theory (HSDT), which is also compared with other HSDTs available in the literature. A practical generalized numerical code for solving the plate governing equations, which can include the shear strain shape functions of existing HSDTs, is utilized. The plate governing equations and boundary conditions are derived by employing the principle of virtual work. Navier-type analytical solution is obtained for FG plates subjected to transverse bi-sinusoidal and distributed loads for simply supported boundary conditions. For the generality of the present HSDT, a continuous isoparametric Lagrangian finite element with 7° of freedom per node are also presented. Results are provided for thick to thin FG plates and for different volume fraction distributions. The accuracy of the present code is verified by comparing it with various HSDTs available in the literature. Results show good agreement between the HSDTs for normal and transversal displacements, normal stresses and in-plane shear stresses. However, opposite occurs for transverse shear stresses. It is because the shear stress results are sensible to the shear strain shape functions used in the formulation of displacement field of a particular HSDT having five unknowns.

1. Introduction

Advanced composite materials such as functionally graded materials (FGMs) have been proposed, developed and successfully used in industrial applications since 1980s [1]. Nowdays FGMs are alternative materials widely used in aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical and shipbuilding industries. Classical composite structures suffer from discontinuity of material properties at the interface of the layers and constituents of the composite. Therefore, stress fields in these regions create interface problems and thermal stress concentrations particularly under high temperature environments. These problems can be decreased by gradually changing the volume fraction of constituent materials, tailoring the material for the desired application. In fact, FGMs are materials with spatial variation of the material properties. However, in most of the applications available in the literature, as in the present work, the variation is through the thickness only. Therefore, the early state development of improved production techniques, new applications, introduction to effective micromechanical models and the development of theoretical methodologies for accurate structural predictions, encourage researchers in this field.

Literature survey shows that many papers dealing with static and dynamic behavior of functionally graded materials (FGMs), have been published recently. An interesting literature review of abovementioned work may be found in the paper of Birman and Byrd [2]. Therefore, for completeness, in the present paper, the relevant work from 2007 up to now is described. For relevant papers perhaps not included in the above mentioned review paper, readers may consult Carrera et al. [3] and Mantari et al. [4].

Zenkour [5] investigated the static problem of transverse load acting on exponentially graded (EG) rectangular plates using both 2D trigonometric plate theory (TPT) and 3D elasticity solution. Sladek et al. [6] presented the static and dynamic analysis of functionally graded plates by the meshless local Petrov–Galerkin method. The Reissner–Mindlin plate bending theory was utilized to describe the plate deformation. Numerical results were presented for simply supported and clamped plates. Bo et al. [7] presented the elasticity solutions for the static analysis of functionally graded plates for different boundary conditions. Stress analysis due to thermal and mechanical loads was given by Matsunaga [8] by using a two-dimensional higher-order theory. A power law distribution for the volume fractions of constituents was assumed for the calculation of modulus of elasticity. Navier solution of a simply supported functionally graded plate was

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provided for stress and deflections. Khabbaz et al. [9] provided a nonlinear solution of FGM plates using the first and third-order shear deformation theories. More recently, Aghdam et al. [10] presented a static analysis of fully clamped functionally graded plates and doubly curved panels by using the extended Kantorovich method. Wu and Li [11] used a RMVT-based third-order shear deformation theory of multilayered FGM plates under mechanical loads. The exponent-law distributions through the thickness and the power-law distributions of the volume fractions of the constituents were used to obtain the effective properties. Talha and Singh [12] investigated the free vibration and static analysis of functionally graded plates using the finite element method by employing a quasi-3D higher order shear deformation theory. Vaghefi et al. [13] presented a three-dimensional static solution for thick functionally graded plates by utilizing a meshless Petrov–Galerkin method. An exponential function was assumed for the variation of Young's modulus through the thickness of the plate, while Poisson's ratio was assumed to be constant. RMVT-based meshless collocation and element-free Galerkin methods for the quasi-3D analysis of multilayered composite and FGM plates were presented by Wu et al. [14]. Carrera et al. [3] studied the effects of thickness stretching in FGM plates and shells. The importance of the transverse normal strain effects in mechanical prediction of stresses for FGM plates was pointed out. In fact, this work is an extension of several FGM paper published by using Carrera's Unified Formulation (CUF), as described in Carrera et al. [15], Brischetto [16] and Brischetto and Carrera [17,18]. Mantari et al. [4] presented bending results of FGM by using a new non-polynomial HSDT, different to the one presented in here. Recently, Neves et al. [19] presented a quasi-3D hybrid polynomial and trigonometric shear deformation theory for the static and free vibration analysis of functionally graded plates by using collocation with radial basis functions.

In the present paper, an analytical solution to the static analysis of functionally graded plates is presented using the recently developed HSDT by Mantari et al. [20], which is compared with the predictions of other HSDTs available in the literature. These HSDTs are: the one which is usually named Touratier's HSDT (well-known trigonometric higher order shear deformation theory which includes sinus function originally developed by Levy [21], corroborated and improved by Stein [22], extensively used by Touratier [23] and recently adapted to FGM and exponentially grade material (EGM) by Zenkour [5,24]); the HSDT presented in Reddy and Liu [25], extended to FG plates in Reddy [26]; the HSDT developed by Soldatos [27]; the HSDT developed by Karama et al. [28,29] which were studied recently by Meichea et al. [30]; and the one developed by Mantari et al. [4]. These theories account for adequate distribution of the transverse shear strains through the plate thickness and tangential stress-free boundary conditions on the plate boundary surface, so that a shear correction factor is not required. The mechanical properties of the panels are assumed to vary in the thickness direction according to a power-law distribution in terms of the volume fractions of the constituents. The governing equations of the functionally graded plate and the boundary conditions are derived by employing the principle of virtual work. These equilibrium equations are then solved using the Navier solution method. Indeed, a practical generalized numerical code for solving the all the plate governing equations (formulated based on the above mentioned HSDTs) is utilized. Bending results are presented for plates for simply supported boundary conditions. FGM plates are subjected to transverse bi-sinusoidal and distributed loads. Results are provided for thick to thin plates and for different volume fraction distributions. The accuracy of the present code is verified by comparing it with various HSDTs available in the literature. In general, results show good agreement between the HSDTs for normal and transversal displacements, normal stresses and in-plane shear stresses. However, opposite occurs for transverse shear stresses.

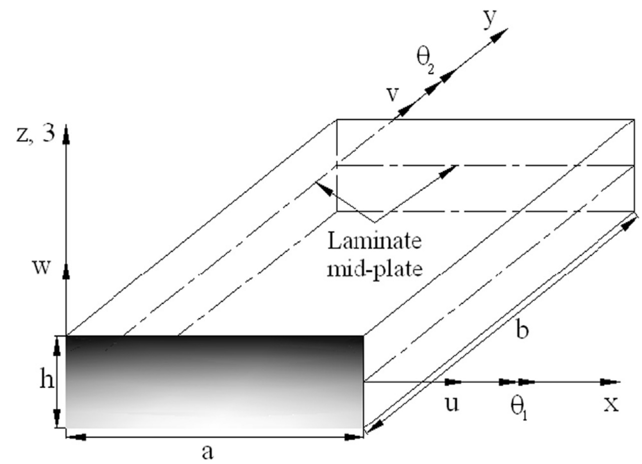

Fig. 1. Geometry of a functionally graded plate.

2. Theoretical formulation

2.1. Functionally graded plates

A rectangular plate of uniform thickness h made of a functionally graded material is shown in Fig. 1. The rectangular Cartesian coordinate system x, y, z, has the plane $z=0$, coinciding with the mid-surface of the plate. The material is inhomogeneous and the material properties vary through the thickness with a simple power-law distribution, which is given below:

$$P(z)=(P_t-P_b)V+P_b, \tag{1a}$$

$$V=\left(\frac{z}{h}+\frac{1}{2}\right)^n, \quad V=g(\overline{z})=\left(\overline{z}+\frac{1}{2}\right)^n. \tag{1b}$$

where P denotes the effective material property, P_t and P_b denote the property of the top and bottom faces of the panel, respectively, and n is the power-law exponent that specifies the material variation profile through the thickness. V, which can be expressed as $g(\overline{z})$, is the volume fraction for different values of n. The effective material properties of the plate, including Young's modulus, E, and shear modulus, G, vary according to Eq. (1a), and Poisson's ratio, ν, is assumed to be constant. It is important to note that n is a parameter that dictates the material variation profile through the plate thickness and takes values greater than zero.

2.2. Displacement field

A recently developed higher order displacement field, in which the displacement of the middle surface is expanded as a combination of exponential and polynomial functions of the thickness coordinate and the transverse displacement taken to be constant through the thickness [20], is considered and it is given below:

$$\overline{u}(x,\ y,\ z)=u(x,\ y)+z\left[y^*\theta_1-\frac{\partial w}{\partial x}\right]+z2.85^{-2(z/h)^2}\theta_1,$$
$$\overline{v}(x,\ y,\ z)=v(x,\ y)+z\left[y^*\theta_2-\frac{\partial w}{\partial y}\right]+z2.85^{-2(z/h)^2}\theta_2,$$
$$\overline{w}(x,\ y,\ z)=w. \tag{2a–c}$$

where $u(x,y)$, $v(x,y)$, $w(x,y)$, $\theta_1(x,y)$ and $\theta_2(x,y)$ are the five unknown displacement functions of middle surface of the panel,