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Damage indices evaluation for seismic resistant structures subjected to low-cycle fatigue phenomena

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ABSTRACT

The study of damage accumulation has a relevant interest in predicting the life of aerospace structures or seismic resistant structures subjected to low-cycle fatigue phenomenon. Damage indices, based on a linear accumulation rule, are presented in this paper. Documented solutions and new proposals for stationary and non-stationary processes are discussed and their accuracy is proved with numerical simulation. These damage indices take into account the randomness of the input excitation applied to the structures. The evaluation of these damage indices requires the knowledge of the statistic of the response process. For a non-stationary process the statistics of the response is calculated through evolutionary power spectral density.

1. Introduction

The damage accumulation due to a random excitation process is an interesting problem to be solved for structures that suffer from fatigue damage such as aerospace structures and seismic resistant structures. In an aerospace structure excitations such as the impact loading on the landing gears of an aircraft and the gust loading may produce damage, this type of excitations are short and transient. The prediction of the fatigue life requires to take into account the non-stationary characteristic of the input excitation. An analogous situation may occur in structures subjected to seismic input excitation. In particular steel structures may suffer damage due to low-cycle fatigue phenomenon that yields an accumulation of damage. This problem has been studied especially for steel structures under seismic actions defying new damage indices accounting strength and stiffness degradation [14], and accounting low cycle fatigue effects in different types of structures [15,8]. This phenomenon has been studied from a deterministic point of view, using linear accumulation law and the commonly used S–N curve as a failure prediction function [11,13,6,4]. But the random characteristic of the seismic input excitation in structures and of transient loading in aerospace structures justifies the attention made in studying the problem from a stochastic point of view.

In this paper some damage indices are proposed to measure the accumulated damage in a structural system due to random excitation process. There are a few works that propose stochastic indices for stationary process [5,17,25], while the search for a solution in the case of non-stationary processes had received less interest, although most of the input excitations are of this type.

A pioneering work was due to Shinozuka and Yang [25], herein their approach is discussed and developed.

The damage accumulation due to a stationary or non-stationary input excitation can be quantified through a damage index, whose definition implies the analysis of the following steps:

- The statistical description of the random input excitation applied to the system;
- The statistical description of the dynamical response of the system;
- The description of fatigue failure mode of the system.

The first step implies a choice between stationary and non-stationary input. In the second step the description of the dynamical characteristic of the system and the probabilistic distribution of the response of the system should be known. In the last step a failure prediction function, and a damage accumulation rule should be established.

In this work the following assumption has been made:

(a) The earthquake load acting on the system is a non-stationary Gaussian process.
(b) System's dynamics is modeled by means of a linear filter.
(c) The fatigue damage is modeled using a suitable S–N curve and Palmgren–Miner damage accumulation rule.

These are common assumptions simplifying the evaluations of fatigue damage indices. Conveniently Gaussianity of loads together

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with linearity of dynamics imply that responses (stresses) are Gaussian processes. Although, this result has been discussed in several papers [1,3], different assumptions on the probabilistic distribution density function of the peaks in the dynamical response due to the applied input yield different damage indices. An overview on the documented damage indices in literature is presented in this paper. To account for non-stationary characteristics of the input, the load has been model as a non-stationary Gaussian load, the non-stationary has been obtained through the evolutionary spectra. A close formula solution to obtain the non-stationary spectra has been proposed. Two damage indices for stationary processes and four damage indices for non-stationary processes are proposed and discussed, finally the accuracy of these indices is proved through simulation.

2. Damage index

A measure of the damage index of a structural system can be computed as soon as the statistical characteristics of the input excitation, structural system response and fatigue endurance of the structure are known. Herein, the *S*–*N* curves are chosen as a failure prediction function [11,13] and expressed in a general form

$$NS^m = C \tag{1}$$

where *S* is the displacement amplitude excursion, *C* and *m* are parameters that depend on the material and on the geometrical characteristics of the system, *N* is the number of cycles to failure due to an applied constant amplitude *S*. This type of failure prediction function is commonly used in high-cycle fatigue, but an extension to low-cycle fatigue has been proposed in literature [9]. Using Miner's rule [18] as a linear damage accumulation rule, the damage at the *i*-cycle d_i due to low-cycle fatigue in a half cycle of amplitude S_i at cycle *i* is

$$d_i = \frac{1}{2C} S_i^m \tag{2}$$

where it is assumed that both peaks and valleys have the same effect on fatigue damage. The damage accumulates linearly and summing the damage at each cycle failure occurs when the accumulated damage reaches an a priori defined value, for instance the unit value.

Denoting the damage in the interval [0 *t*]′ with $D(t)$ and *N* the number of peaks in the same interval, the expected damage $E[D(T)|N=n]$ due to exactly *n* cycles in $t=T$ is given by

$$E[D(T)|N=n] = E\left[\frac{1}{2C}\sum_{i=1}^{2N}|S_i^m|\right] \tag{3}$$

where *E*[.] is the expectation function. The *S*–*N* curve is not only expressed in terms of stress amplitude, but also in terms of strain amplitude or global coordinates such as deformation, rotation or moment [13]. In any case, the stress amplitude *S* is related to any global coordinates by a linear relationship of type

$$S = C_s x \tag{4}$$

If the *S*–*N* curve, obtained by the analysis of experimental test result on the cantilever beam, is expressed in terms of maximum displacement excursion at the top of a cantilever beam in Fig. 1, then the constant C_s is equal to 2. Hence, substituting Eq. (4) in Eq. (3), the expected damage for *n* cycles at time *t* can be given in its integral form [25,17,23]

$$E[D(T)/N=n] = \frac{nC_s^m}{C}\int_0^\infty x^m f_P(x,T)dx \tag{5}$$

where $f_p(x,T)$ is the probability density function of peaks of the response process $x(t)$ in the interval *T*.

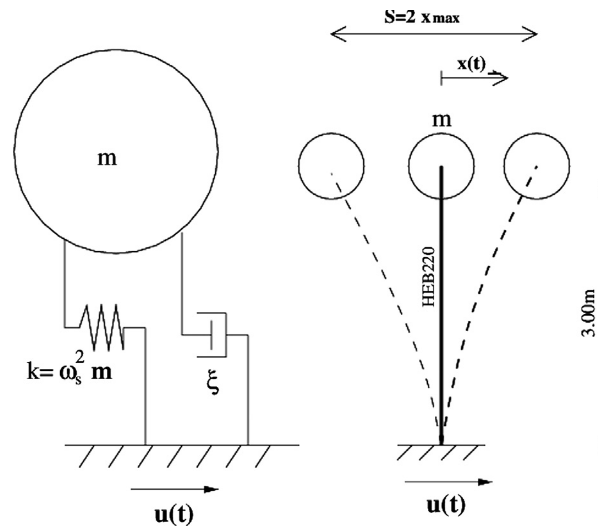

Fig. 1. SDOF system.

The expected damage is

$$E[D(T)] = \sum_{n=1}^{\infty} E[D(T)|N=n]Pr(N=n) = \frac{\overline{N}C_s^m}{C}\int_0^\infty x^m f_P(x,T)dx \tag{6}$$

where $\overline{N}$ is the expected number of zero-level crossings in the history $x(t)$ in [0, *T*]. The crucial point is the knowledge of the probability density function of the peaks $f_p(x,T)$. Different choices on probability density functions of peak response process $f_p(x,T)$ yield different damage indices.

3. The engineering approach to fatigue damage assessment

The first documented solution for the damage index, due to Rice [23] later developed by Miles [17], is only applicable to a narrow band stationary process. Successively Bendat in 1964 [2] demonstrated that the narrow band approximation is not introduced through the concept of envelope but from Rice's formula for expected number of crossings, in that sense it is not limited to Gaussian loads.

The response of the SDOF system (see Fig. 1) subjected to a stationary input is such a narrow band process. The distribution of the peaks x_p is a Rayleigh distribution:

$$Pr(x < x_P < x+dx) = f_P dx = \frac{x\exp[-x^2/(2\sigma_x)]dx}{\sigma_x^2} \tag{7}$$

This result is explained by Miles for lightly damped systems. Miles [13], following an interpretation by Rice [19] on the envelope function of the harmonic waves with random amplitudes, introduced the concept that synthesis of two harmonic waves that have approximately the same frequency produces oscillatory waves with a frequency equal to the mean of the two frequencies and with an envelope amplitude changing over the frequencies. They also found that in this case the envelope process composed of a large number of harmonics has a frequency in the range $(\omega_o \pm \delta\omega_o)$ and a random phase.

Suppose the distribution of peaks $f_p(x,t)$ of the stationary response is a Rayleigh distribution as in Eq. (7), Eq. (6) can be integrated on time and a close form expression is obtained for the damage index I_D [5,17,23,22] at time $t=T$ as in the following