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Moment distribution around polygonal holes in infinite plate

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ABSTRACT

General solutions for determining the moment distribution around polygonal holes in infinite isotropic plate subjected to bending/twisting moment at infinity are obtained using Muskhelishvili's complex variable method. The conformal mapping and biaxial loading factor is introduced to take care hole geometry and loading conditions.

The generalized formulation thus obtained is coded and numerical results are obtained for triangular, square, pentagonal, hexagonal, heptagonal and octagonal cut-outs. The effect of hole geometry and loading pattern on moment distribution is studied.

1. Introduction

The different shaped cut-outs are made in mechanical, civil and aero-structures to satisfy certain service requirements. These cut-outs reduce strength of the structures and may lead to failure. It is essential to study the behavior of plate type of structures with cut-outs under the application of different types of loading.

The research on the stress distribution around hole is ongoing since first solution for stress distribution around circular hole is presented by Kirsch [1] using real variables. Savin [2], Lekhnitskii [3], Daoust and Hoa [4], Ukadgaonker and Rao [5,6], Ukadgaonker and Awasare [7], Theocaris and Petrou [8], Ukadgaonker and Kakhandki [9], Gao [10], Rezaeepazhand and Jafari [11], Batista [12], Sharma [13,14], Rao et al [15], etc. presented solutions for plate with different shaped holes, subjected to in-plane tensile loads (uniaxial/biaxial) at infinity. Using Muskhelishvili's [16] complex variable method, Savin [2] and Lekhnitskii [3] presented stress distribution around circular, elliptical, triangular, square and rectangular holes in isotropic plates. The formulations for stress field around triangular [4, 5,8, 11, 13], rectangular [15], polygonal [12,14], irregular shaped [6, 9] holes are available and present effect of hole geometry, material parameters and loading patterns on stress field.

The moments around circular and elliptical hole are determined for remotely applied cylindrical bending, all-round bending and twisting by Goodier [17] based on the thin plate theory. He used method of superposition for solution of all-round bending and twisting moment problem. Using principle of stationary potential energy, Chen and Archer [18] gave the solution to determine the stress concentration factor and stress couple concentration factor around a circular hole in thick plate. The moment distribution around circular, elliptical, triangular and square hole in infinite isotropic plate subjected to bending is obtained by Savin [2] using complex variable approach. Ukadgaonker and Rao [19] developed generalized formulation for bending and twisting of symmetric laminate. Gao's [10] loading condition and Savin's [2] integro-differential formulation based on Muskhelishvili's [16] classical work are employed to study bending and twisting of laminated composites.

Seeing through the literature, it is evident that very few solutions are available for bending of plates with holes, particularly special shaped holes. As per the best of author's knowledge, the solution for moment distribution around polygonal hole using complex variable is not reported in the literature.

In this paper generalized solution for determining moment distribution around polygonal hole (triangular, square, pentagonal, hexagonal, heptagonal and octagonal) in an infinite plate, under cylindrical, all-round and twisting moments is presented using complex variable method. The effect of hole geometry and loading pattern on moment distribution is studied.

2. Mapping function

In order to find moment distribution around a hole in z-plane, the area outside the polygonal hole is mapped to a region outside unit circle in ζ-plane, which has an origin at $\zeta=0$ using following

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Schwartz-Christoffel type mapping function:

$$z=\omega(\zeta)=R\left(\zeta+\sum_{k=1}^{m}\frac{c_k}{\zeta^{kn-1}}\right) \tag{1}$$

where

$$c_1=\left(\frac{2}{n(n-1)}\right),\quad c_2=\left(\frac{n-2}{n^2(2n-1)}\right),$$
$$c_3=\left(\frac{(n-2)(2n-2)}{3n^3(3n-1)}\right),\quad c_4=\left(\frac{(n-2)(2n-2)(3n-2)}{12n^4(4n-1)}\right)\ldots$$

R is a constant for size of hole, n is a number of sides of polygon.

3. Stress functions

Fig. 1 shows a plate with square hole, subjected to moment about arbitrary axis x'–y' rotated by an angle β from x–y-axis. The moments applied at infinity about arbitrary axis x'–y' are $M_{x}{'}=\lambda M$ and $M_{y}{'}=M$ (λ=biaxial loading factor [10]).

On the basis of 'hypothesis of straight normal', the displacements u and v in x and y directions and the corresponding strains ε_x, ε_y and γ_{xy} are expressed as functions of deflection $w(x,y)$ of the mid-plane in the z-direction as

$$u=-z\frac{\partial w}{\partial x},\quad v=-z\frac{\partial w}{\partial y} \tag{2}$$

$$\varepsilon_x=-z\frac{\partial^2 w}{\partial x^2},\quad \varepsilon_y=-z\frac{\partial^2 w}{\partial y^2},\quad \gamma_{xy}=-2z\frac{\partial^2 w}{\partial x\partial y} \tag{3}$$

For the thin plates having thickness h (plane stress conditions), the stress–displacement relation can be written as follows:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\frac{-E}{1-\upsilon^2}\begin{bmatrix}1 & \upsilon & 0\\ \upsilon & 1 & 0\\ 0 & 0 & \frac{1-\upsilon}{2}\end{bmatrix}\begin{Bmatrix}z\frac{\partial^2 w}{\partial x^2}\\ z\frac{\partial^2 w}{\partial y^2}\\ 2z\frac{\partial^2 w}{\partial x\partial y}\end{Bmatrix} \tag{4}$$

where

E is a modulus of elasticity, ν is Poisson's ratio

Integrating stresses after multiplying by z in the limits of $-h/2$ to $h/2$, we get moments M_x, M_y and M_{xy} as

$$\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}z\,dz=-D\begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\\ \frac{\partial^2 w}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}\\ (1-\upsilon)\frac{\partial^2 w}{\partial x\partial x}\end{Bmatrix} \tag{5}$$

$D=Eh^3/12(1-\nu^2)$ is the cylindrical rigidity.

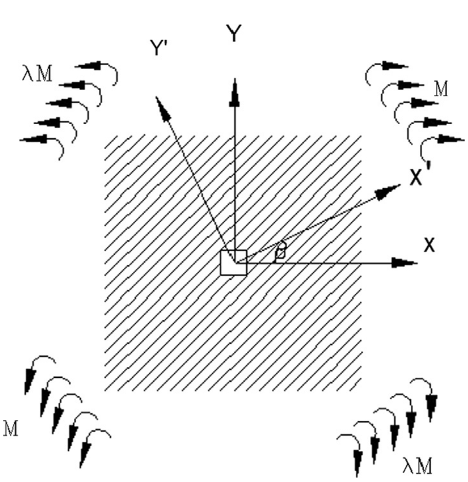

Fig. 1. Loading on the plate with square hole.

By taking the general equation of deflection in terms of arbitrary analytic function $\phi(z)$ and $\psi(z)$ as

$$w(x,y)=\mathrm{Re}\left(z\phi(z)+\int\psi(z)dz\right) \tag{6}$$

The basic equations of plane elasticity in complex variable form are given by Muskhelishvilli [16] as follows:

$$M_x+M_y=-2D(1+\upsilon)[\phi'(z)+\overline{\phi'(z)}] \tag{7}$$

$$M_y-M_x+2iM_{xy}=2D(1-\upsilon)[\bar{z}\phi''(z)+\psi'(z)] \tag{8}$$

where

$\phi(z)$, $\psi(z)$ is a complex potentials of the complex variable $z=x+iy$.
$\phi'(z)$, $\psi'(z)$ is a first derivative of the complex potentials.
$\phi''(z)$, $\psi''(z)$ is a second derivative of the complex potentials.

In the absence of hole, Eqs. (7) and (8) can be written in terms applied moments at infinity:

$$M_x^\infty+M_y^\infty=M(\lambda+1)=-2D(1+\upsilon)[\phi_0'(z)+\overline{\phi_0'(z)}] \tag{9}$$

$$M_y^\infty-M_x^\infty+2iM_{xy}^\infty=Me^{-2i\beta}[1-\lambda]=2D(1-\upsilon)[\bar{z}\phi_0''(z)+\psi_0'(z)] \tag{10}$$

Solving Eqs. (9) and (10), we get

$$\phi_0'(z)=\frac{-(1+\lambda)M}{4D(1+\upsilon)} \tag{11}$$

$$\psi_0'(z)=\frac{(1-\lambda)Me^{-2i\beta}}{2D(1-\upsilon)} \tag{12}$$

By multiplying Eqs. (11) and (12) by $\omega'(\zeta)$ and then integrating, we get

$$\phi_0(\zeta)=k_1\omega(\zeta) \tag{13}$$

$$\psi_0(\zeta)=k_2e^{-2i\beta}\omega(\zeta) \tag{14}$$

where

$$k_1=\frac{-(1+\lambda)M}{4D(1+\upsilon)},\quad k_2=\frac{(1-\lambda)M}{2D(1-\upsilon)}$$

The boundary conditions imposed by this stress functions on hole surface can be written as

$$f=N\phi_0(z)+z\overline{\phi_o'(z)}+\overline{\psi_0(z)} \tag{15}$$

By introducing stress functions in above equation the boundary conditions can be represented as

$$f(t)=(N+1)k_1R\left\{t+\sum_{k=1}^{m}\frac{c_k}{t^{kn-1}}\right\}+k_2e^{-2i\beta}\left\{\frac{1}{t}+\sum_{k=1}^{m}c_kt^{kn-1}\right\} \tag{16}$$

($\zeta=t$ at hole surface).

The stress functions $\phi_1(\zeta)$ and $\psi_1(\zeta)$ can be obtained by evaluating Cauchy's integral:

$$\phi_1(\zeta)=-\frac{1}{2\pi i}\oint\frac{f(t)dt}{t-\zeta} \tag{17}$$

$$\psi_1(\zeta)=-\frac{1}{2\pi i}\oint\frac{\overline{f(t)}dt}{t-\zeta}-\frac{\overline{\omega(\zeta)}}{\omega'(\zeta)}\phi_0'(\zeta) \tag{18}$$