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Plastic flow localization analysis of heterogeneous materials using homogenization-based finite element method

Yuichi Tadano [a,*], Kengo Yoshida [b], Mitsutoshi Kuroda [b]

[a] Saga University, 1 Honjo-machi, Saga 840-8502, Japan
[b] Yamagata University, 4-3-16 Jonan, Yonezawa, Yamagata 992-8510, Japan

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A novel framework to predict the onset of plastic flow localization is presented. The proposed framework combines a classical strain localization analysis with a homogenization-based finite element method, and has high applicability to various types of material with a characteristic microstructure that may have significant heterogeneity as long as its representative volume element can be represented by a finite element discretization. According to the proposed method, a plastic flow localization analysis can be performed taking only one or two material points in macroscopic analysis. This means that localization analysis of materials involving very complex microstructures, which is hard to be satisfactorily treated in a fully micro-macro-coupled finite element analysis with the homogenization approach, can be carried out with a reasonable computational cost. As a practical application of the proposed general framework, a plane stress version, that is, a Marciniak–Kuczyński-type (M–K) approach, is considered, then the forming limit strains of FCC polycrystalline sheets are evaluated. Crystal plasticity theory is adopted as a constitutive model for each crystal grain, and the homogenization-based finite element method is used to evaluate the average material response to be used in M–K-type sheet necking analysis. A numerical convergence analysis is conducted to determine the appropriate size of the representative volume element in the homogenization, and the effect of the geometrical configuration of crystal grains is studied. Then, the forming limit strains of a textured material are evaluated. The computational results are compared with those obtained using the conventional Taylor-type polycrystalline model.

1. Introduction

Prediction of the ductile fracture of materials is a very important issue in structural engineering as well as in the metal forming field. The localization of plastic deformation is a precursor of rupture in ductile materials. In three-dimensional bulk materials, plastic flow localization appears in the form of *shear bands*. In sheet materials, it occurs in the form of narrow zones with significant reduction of the thickness, which is often modeled as *sheet necking* under the assumption of plane stress. In general scenarios in engineering, localization occurs in nonhomogeneous deformation fields where stress and strain gradients exist caused by holes, notches, or other types of geometric or material nonuniformity. In such cases, a full boundary value problem solution is required to predict the development of plastic flow localization. Owing to the fact that the phenomena themselves are strongly history-dependent and nonlinear, a powerful numerical method such as the finite element method is usually required to investigate such material behavior. However, the whole deformation history up to the final rupture generally involves an abrupt change from an initial deformation pattern to the subsequent pattern. If we assume that the deformation pattern in the pre-localization stage is homogeneous and the material is rate-independent, a bifurcation analysis can be applied to determine the critical state at which the deformation pattern changes from the homogeneous mode to a highly concentrated mode of shear bands or sheet necking [1,2]. This bifurcation analysis requires only a *single material point* in order to detect the critical state for the onset of localization. For rate-dependent solids, into which most real materials are classified, a bifurcation analysis is not relevant because the critical state is governed by elastic properties [3]. Instead, an imperfection-based analysis is useful, in which a material or geometric imperfection evolves with deformation and localization is considered to occur when plastic straining continues only within the imperfection zone while strain rate reversal occurs outside the imperfection zone. In this type of imperfection analysis, only two material points are required to detect the onset of localization. Moreover, the post-localization behavior can be analyzed beyond the critical point. A general theoretical framework and some specific formulations of computations for bifurcation and imperfection analyses were

* Corresponding author. Tel.: +81 952 28 8483; fax: +81 952 28 8587.
E-mail address: tadano@me.saga-u.ac.jp (Y. Tadano).

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reviewed and presented by Needleman and Tvergaard [1] and Rice [2].

In the sheet-metal-forming field, the analysis of plastic flow localization is particularly important since the limits to the ductility (i.e., formability) of sheet materials are set by the occurrence of *sheet necking*. The Marciniak and Kuczyński [4] (henceforth referred to as M–K) type approach, which is a kind of imperfection analysis, has been extensively employed. The M–K type approach is considered to be one of the most influential contributions of flow localization analysis to the practical metal-forming field. Of course, the forming limit strains of a sheet specimen strongly depend on the material model employed [5]. Basically, any kind of constitutive model can be applied to the M–K type approach. A number of computations of forming limit strains based on the M–K type approach with phenomenological constitutive models composed of different types of yield function and flow rule have been extensively conducted (e.g., [6–11]). In the case of polycrystalline metals, the crystallographic microstructure such as the texture, which leads to plastic anisotropy, strongly affects the forming limit strains. Although the behavior of each crystal grain might be well described by the crystal plasticity theory, relevant methods of evaluating the average response of a polycrystalline aggregate are still an issue. Several studies have employed a Taylor-type approach as a simplified homogenization method, which assumes that all grains undergo the same deformation history [12–17]. However, it has been pointed out that the Taylor model provides a stiffer material response due to the strong constraint on the deformation of crystal grains [18]. Another idea for reproducing the polycrystalline behavior is by a self-consistent approach [19]. Studies of forming limit strains obtained using the viscoplastic self-consistent (VPSC) model have been reported in recent years (e.g., [20–22]). Despite a number of these studies, it has not been revealed which approach is the most relevant for representing the averaged material behavior of polycrystalline metals. Even the limitation of the classical Taylor model, the simplest approach for modeling the polycrystal response, has not yet been clarified. For materials that have a more complex microstructure, such as composite, multiphase, or damaged materials, no general method of evaluating a macroscopic material property originating from their specific microstructure in the plastic flow localization analysis has been established.

A modern powerful method of describing the macroscopic behavior of a microscopically heterogeneous material has been proposed by Guedes and Kikuchi [23]. This method is based on the two-scale asymptotic expansion of field variables such as displacement and can be used to evaluate homogenized macroscopic material properties. An important advantage of this homogenization approach is that a macroscopic constitutive relation for the material can be automatically obtained from its microstructure information. As an example of the application of the homogenization method to practical metals, Nakamachi et al. [24] applied a finite element polycrystalline plasticity analysis with the homogenization method to sheet-metal-forming simulations with crystallographic texture information. The present authors also used the homogenization crystal plasticity model (HOM-CP) to evaluate the macroscopic material behaviors of a material with a crystalline aggregate [18,25,26]. The HOM-CP is expected to take account of realistic deformation behavior of each grain in a crystal aggregate and to easily handle macroscopic boundary conditions. On the other hand, the homogenization approach generally has a high computational cost since it requires a large-scale numerical computation by the finite element method. This is a critical disadvantage of the homogenization-based finite element method, particularly in the case of calculating full boundary value problems, in which the homogenization calculations must be performed at every integration point at every increment in the nonlinear plasticity analysis. For instance, a finite element analysis, in which the localized deformation such as the shear band occurs, generally requires a large number of finite elements in the macroscopic analysis. As a result, the computation of macroscopic plastic flow localization phenomena taking into account the full interaction between the microstructure and macrostructure may become a formidable task in general cases. Consequently, a microstructure that is complex in reality is often modeled with an unsatisfactorily small number of finite elements in a fully micro-macro-coupled analysis in practice.

In this way, the homogenization-based finite element analysis of the whole domain of a specimen is generally impractical. On the other hand, as mentioned above, bifurcation analysis to detect the onset of localization requires only a single material point, and imperfection analysis requires only two material points. With this background, a combination of bifurcation- or imperfection-type analysis and the homogenization scheme proposed by Guedes and Kikuchi [23] would appear to be an efficient method for predicting plastic flow localization from the viewpoints of both physical relevance and computational cost. In the present paper, we propose a framework to predict plastic flow localization phenomena utilizing the homogenization method. As a practical application to demonstrate the potential of the proposed general framework, the M–K type approach and the HOM-CP are combined, and forming limit strains for FCC polycrystals are computed. A numerical convergence analysis is conducted to determine an appropriate size for the representative volume element (RVE) in the homogenization-based finite element method. Then, the effect of the geometrical configuration of crystal grains is investigated. Finally, the forming limit strains of a cube-textured material are evaluated. In all computations, results are compared with those obtained with the Taylor model, and the validity of using the Taylor model in forming limit analysis is also discussed.

It is emphasized that this paper discusses the adequacy of presented framework and the combination of the HOM-CP with the M–K approach is an example within the proposed methodology. Utilizing the present framework, it will be possible to perform three-dimensional shear band analysis, but this is left for a future study.

2. Illustration of the proposed method

2.1. General framework

Localized plastic flow in the form of shear bands tends to occur rather abruptly from a smoothly varying deformation [3]. The basic phenomenon of shear band formation can be studied by considering a relatively simple model (e.g., [3,27]). Fig. 1 shows a general illustration of the problem. A bulk specimen includes a band (thin slice) with an initial inhomogeneity with unit normal $\mathbf{n}$, and the strain fields outside the band are assumed to always be uniform. The initial direction of $\mathbf{n}$ is denoted by $\mathbf{n}_\mathrm{I}$, which is specified in terms of the initial angle ψ_I between the normal $\mathbf{n}_\mathrm{I}$ and the x_1–x_3 plane and the initial angle ϕ_I between the x_1 axis and the projection of $\mathbf{n}_\mathrm{I}$ on the x_1–x_3 plane. The quantities inside and outside the band are denoted by $(\,)^\mathrm{b}$ and $(\,)^\mathrm{o}$, respectively. The compatibility and equilibrium conditions at the band interface are

$$\mathbf{L}^\mathrm{b} = \mathbf{L}^\mathrm{o} + \dot{\mathbf{c}} \otimes \mathbf{n} \tag{1}$$

$$\mathbf{n} \cdot \bar{\boldsymbol{\sigma}}^\mathrm{b} = \mathbf{n} \cdot \bar{\boldsymbol{\sigma}}^\mathrm{o}, \tag{2}$$

where $\mathbf{L}$ is the velocity gradient tensor and $\bar{\boldsymbol{\sigma}}$ is the homogenized Cauchy stress tensor. $\dot{\mathbf{c}}$ is a vector parameter to be determined as a solution to this problem. The general constitutive equation for rate-dependent materials is given as

$$\dot{\bar{\boldsymbol{\sigma}}} = \tilde{\bar{\mathbf{C}}} : \mathbf{L} - \dot{\bar{\mathbf{P}}}, \tag{3}$$