International Journal of Mechanical Sciences 72 (2013) 75–79

Contents lists available at SciVerse ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

A model for the contact behaviour of weakly orthotropic viscoelastic materials

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ARTICLE INFO

Article history:
Received 16 March 2012
Received in revised form
20 March 2013
Accepted 27 March 2013
Available online 10 April 2013

Keywords:
Contact mechanics
Orthotropic material
Anisotropy
Viscoelastic material
Fibre-reinforced elastomer

ABSTRACT

Fibre reinforced elastomers behave anisotropically as well as viscoelastically. Yang and Sun (1982) developed an elastic contact model for anisotropic materials which, in the present work, is extended to account for viscoelastic effects. The developed viscoelastic contact model uses the creep compliance function of the material in the direction of indentation. The results of the model agree with experimental results obtained on short fibre reinforced EPDM. Furthermore, a parameter study of the coefficients of the creep compliance function on the real contact area has been made. The results show that, at short time scales, the viscoelastic real area of contact can be significantly smaller than when assuming fully elastic behaviour. At long time scales the results of the viscoelastic contact model equal those of the elastic model of Yang and Sun.

1. Introduction

Fibre reinforced elastomers are used in a range of industrial applications such as tyres, transmission belts and seals. Similar to fibre reinforced polymers, the addition of fibres to an elastomeric matrix causes improved mechanical performance, albeit typical reinforcements for elastomers, especially those with short fibres, are rather low [1,2]. With the increasing use of short fibre reinforced elastomers, it has become important to characterize their contact behaviour as this influences their tribological performance. Fibre reinforced elastomers behave viscoelastically due to the elastomeric matrix and anisotropically as the result of the preferred orientation of the fibres. To determine the contact between a rigid spherical indenter and an anisotropic viscoelastic material we require a contact model that considers both effects.

Models describing the contact behaviour of viscoelastic materials are usually limited to isotropic material behaviour. In the present study, a viscoelastic anisotropic material with a low degree of anisotropy is considered. This means that the time dependent material properties, such as creep compliance in tensile and shear, have similar, but not necessarily equal values in each principal direction.

The effect of anisotropy in the elastic contact problem was studied theoretically by Willis [3], who considered a three dimensional elastic contact of full anisotropic bodies. Swanson [4] used the Willis [3] approach to calculate the Hertzian contact problem for elastic orthotropic materials. Therefore, if provided with nine different material properties (such as the elastic moduli, shear moduli and Poisson's ratios, in three directions) it is possible to calculate the area of contact for orthotropic materials. However, for fibre reinforced elastomers in the rubbery state, it is difficult to accurately obtain the material properties in that many directions and a model that describes the contact behaviour whilst requiring a smaller number of parameters is highly desirable.

Yang and Sun [6] and Tan and Sun [7] performed indentation tests in laminated composites, showing that the loading curve for these anisotropic materials follows a power law with the same index as would be expected from the Hertz theory. Consequently, for an anisotropic elastic material that is indented by a rigid sphere they proposed to approximate the deformation by using the elastic modulus of the anisotropic material in the direction of indentation only.

Chen [8] showed that for isotropic and anisotropic materials under pure normal loading, the normal displacements and the pressure distributions are identical; only the absolute values of the pressure may differ. Furthermore, according to Chen [8], the stress distribution inside an elastically deforming orthotropic body is symmetrical when one of the principal axes of the orthotropic material coincides with the direction of indentation.

This means that the approximation of Yang and Sun [6] and Tan and Sun [7] to model the anisotropic behaviour as a modification of the isotropic behaviour appears to be valid.

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http://dx.doi.org/10.1016/j.ijmecsci.2013.03.016

Nomenclature

a	radius of the contact area, for elastic materials [mm]
$a(t)$	radius of the contact area, for viscoelastic materials [mm]
E'	equivalent elastic modulus [MPa]
E_z	elastic modulus in the direction of indentation [MPa]
E_x	elastic modulus perpendicular to the indentation direction [MPa]
F_N	applied normal force [N]
$F_N(t)$	time dependent applied normal force [N]
$H(t)$	heaviside function [–]
$p(r,t)$	pressure distribution at a viscoelastic contact [MPa]
r	spatial variable in the contact pressure [mm]
R	radius of the spherical indenter [mm]
S^*	time it takes to reach a stable state in viscoelastic contact [s]
t	time, temporal variable in the contact pressure [s]
$\psi_z(t)$	stress relaxation function, measured in the indentation direction [MPa^{-1}]
$\phi_z(t)$	creep compliance function, measured in the indentation direction [MPa^{-1}]
$\phi\ (t)$	creep compliance function of the material model [MPa^{-1}]
ϕ_r	relaxed creep compliance [MPa^{-1}]
λ_i	retardation time of the ith component [m s]

The above is valid for purely elastic materials. Viscoelastic materials show time dependent behaviour which affects the contact problem, as the boundary conditions change to become time dependent functions. This type of problem is more complex to solve than a time independent problem because methods such as the Laplace transform and the elastic–viscoelastic correspondence principle cannot be directly applied. A solution to the viscoelastic isotropic contact problem was found by Lee and Radok [9], who obtained the pressure distribution over the contact area for a non-decreasing contact area function. Graham [10] obtained viscoelastic analogues to the Hertz equations for a contact area function with a single maximum and, later, for a time dependent contact area with any number of maxima or minima [11]. Ting [12,13] expressed the viscoelastic solutions in terms of the solution to the elastic contact problem, thus solving the contact stresses between a viscoelastic solid and a rigid indenter for a contact area that follows an arbitrary time function.

The viscoelastic contact models discussed in [9–13] are for homogeneous and isotropic materials. In the present study, the method of Yang and Sun [6] and Tan and Sun [7] that approximates orthotropic behaviour by using the material properties in only one direction is extended to consider viscoelasticity. This is done by replacing the isotropic viscoelastic time function by the anisotropic viscoelastic time function in the direction of indentation.

2. Sun's anisotropic contact model

As discussed before, Yang and Sun [6] and Tan and Sun [7] proposed an approximation for the deformation of an anisotropic elastic material that is indented by a rigid sphere. Following this approximation, the contact area is a circle of radius given by

$$a = \left(\frac{3RF_N}{4E'_z}\right)^{1/3} \tag{1}$$

where E'_z is the reduced elastic modulus in the direction of indentation, z. A comparison of the contact areas calculated employing this unidirectional model with Willis's anisotropic contact model [3] has been made by Swanson [4]. He found that the anisotropic contact model only gives a 4% larger contact area than the unidirectional model, for a material that is twice as stiff in the plane of indentation (i.e. perpendicular to the indentation direction). Therefore the approximation of Yang and Sun [6] and Tan and Sun [7] can be considered valid for materials with a low degree of anisotropy. At high degrees of anisotropy, say when the difference in properties is more than 400%, the approximation proposed by Yang and Sun [6] and Tan and Sun [7] is not valid [4,5].

3. Extension to viscoelastic contact, based on Sun's model

A common approach in linear viscoelastic theory, see e.g. Lee and Radok [9] and Ting [12], is to express the viscoelastic solution of the isotropic contact problem in terms of the elastic solution. Following this thought, the stress distribution inside an orthotropic viscoelastic material can be described based on the orthotropic elastic solution. This means that, for an anisotropic viscoelastic material with a low ratio of reinforcement (i.e. $E_z/E_x < 2$), the time dependent contact area can be calculated by combining the solution of the viscoelastic contact problem with the elastic model described in [6,7]. This means that the contact behaviour of the viscoelastic anisotropic material is characterised by time dependent material properties measured in the direction of indentation, such as a stress relaxation function $\psi_z(t)$ or a creep compliance function $\phi_z(t)$.

For the loading phase of the contact, i.e. for an increasing contact area, the pressure distribution in the contact area, p(r,t) is given by

$$p(r,t) = \frac{4}{\pi R}\int_0^t \psi_z(t-\tau)\frac{d}{dt}\left\{a^2(\tau)-r^2\right\}^{1/2} d\tau \tag{2}$$

where R is the radius of the spherical indenter, a is the radius of the contact area, r and t are the spatial and temporal variables, respectively and τ is the dummy variable from the convolution integral.

The total applied time dependent normal force, $F_N(t)$, is calculated by integrating the pressure distribution over the contact area. This results in

$$F_N(t) = \frac{8}{3R}\int_0^\infty \psi_z(t-\tau)\frac{\partial}{\partial\tau}(a^3(\tau))d\tau \tag{3}$$

The creep compliance $\phi_z(t)$ and stress relaxation $\psi_z(t)$ functions are related in the Laplace domain according to

$$\psi_z(s)\phi_z(s) = \frac{1}{s^2} \tag{4}$$

Assuming that the force is defined by a Heaviside function as $F_N\ (t) = H(t)\cdot F_N$, Eq. (3) can be inverted, giving the radius of the contact area:

$$a^3(t) = \frac{3}{8}R\cdot F_N\cdot\int_0^t \phi_z(t-\tau)\frac{d}{d\tau}H(\tau)d\tau \tag{5}$$

Solving the integral allows to express the radius of the viscoelastic contact area as

$$a(t) = \left(\frac{3R\cdot F_N}{8}\phi_z(t)\right)^{1/3} \tag{6}$$