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Static analysis of laminated composite curved shells and panels of revolution with a posteriori shear and normal stress recovery using generalized differential quadrature method

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ABSTRACT

The Generalized Differential Quadrature (GDQ) Method is applied to study laminated composite shells and panels of revolution. The mechanical model is based on the so called First-order Shear Deformation Theory (FSDT) deduced from the three-dimensional theory, in order to analyze the above moderately thick structural elements. In order to include the effect of the initial curvature from the beginning of the theory formulation a generalization of the kinematical model is adopted for the Reissner–Mindlin theory. The solution is given in terms of generalized displacement components of points lying on the middle surface of the shell. The results are obtained taking the two co-ordinates into account, without using the Fourier expansion methodology, as done in semi-analytical methods. After the solution of the fundamental system of equations in terms of displacements and rotations, the generalized strains and stress resultants can be evaluated by applying the Differential Quadrature rule to the generalized displacements themselves. The transverse shear and normal stress profiles through the laminate thickness are reconstructed a posteriori by simply using local three-dimensional equilibrium equations. No preliminary recovery or regularization procedure on the extensional and flexural strain fields is needed when the Differential Quadrature technique is used. By using GDQ procedure through the thickness, the reconstruction procedure needs only to be corrected to properly account for the boundary equilibrium conditions. In order to verify the accuracy of the present method, GDQ results are compared with the ones obtained with 3D finite element methods. Stresses of several composite shell panels are evaluated. Very good agreement is observed without using mixed formulations and higher order kinematical models. Various examples of stress profiles for different shell elements are presented to illustrate the validity and the accuracy of GDQ method.

1. Introduction

Shells have been widespread in many fields of engineering as they give rise to optimum conditions for dynamic behavior, strength and stability. The deflection and interlaminar state of stress on these structures caused by different forces can have serious consequences for their strength and safety. Therefore, an accurate deflection and interlaminar state of stress determination are of considerable importance for the technical design of these structural elements. The aim of this paper is to study the static behavior of shell structures, which are very common structural elements.

During the last 60 years, two-dimensional linear theories of thin shells and plates have been developed including important contributions by Timoshenko and Woinowsky-Krieger [1], Flügge [2], Gol'Denveizer [3], Novozhilov [4], Vlasov [5], Ambartusumyan [6], Kraus [7], Leissa [8,9], Markuš [10], Ventsel and Krauthammer [11] and Soedel [12]. All these contributions are based on the Kirchhoff–Love assumptions. This theory, named Classical Shell Theory (CST), assumes that normals to the shell middle-surface remain straight and normal to it during deformations and unstretched in length.

When the theories of thin shells are applied to thick shells, the errors could be quite large. With the increasing use of thick shells in various engineering applications, simple and accurate theories for thick shells have been developed. With respect to thin shells, thick shell theories take the transverse shear deformation and rotary inertia into account. The transverse shear deformation has been incorporated into shell theories by following the theory of Reissner–Mindlin [13], also named First-order Shear Deformation Theory (FSDT). Abandoning the assumption on the preservation of the normals to the shell middle surface after the deformation, a

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Nomenclature

$O'\varphi s\zeta$ local shell co-ordinate system
$Ox_1x_2x_3$ global co-ordinate system
x'_3 geometric axis of the meridian curve
l number of laminae
h thickness of the shell
h_k thickness of the kth lamina
ζ_k bottom co-ordinate of the kth lamina
ζ_{k+1} top co-ordinate of the kth lamina
ϑ circumferential angle
R_b shift of the geometric axis of the curved meridian x'_3
R_0 circumferential radius
R_φ radius of curvature along the meridian φ co-ordinate
R_s radius of curvature along the circumferential s co-ordinate
U_φ, U_s, U_ζ 3D shell displacement components
u_φ, u_s, u_ζ displacement components of points lying on the middle surface ($\zeta=0$) of the shell
β_φ, β_s normal-to-mid-surface rotations
$\varepsilon^0_\varphi, \varepsilon^0_s, \gamma^0_\varphi, \gamma^0_s$ in-plane meridian, circumferential and shearing components
$\chi^0_\varphi, \chi^0_s, \omega^0_\varphi, \omega^0_s$ meridian, circumferential and torsional curvature changes
$\gamma^0_{\varphi n}, \gamma^0_{sn}$ transverse shearing strains
$\overline{Q}^{(k)}_{pq}, \overline{Q}^{(k)}_{11}, \overline{Q}^{(k)}_{12}, \overline{Q}^{(k)}_{16}, \overline{Q}^{(k)}_{22}, \overline{Q}^{(k)}_{26}, \overline{Q}^{(k)}_{66}, \overline{Q}^{(k)}_{44}, \overline{Q}^{(k)}_{45}, \overline{Q}^{(k)}_{55}$ transformed plane stress-reduced stiffnesses
$\overline{C}^{(m)}_{11}, \overline{C}^{(m)}_{12}, \overline{C}^{(m)}_{13}, \overline{C}^{(m)}_{16}, \overline{C}^{(m)}_{22}, \overline{C}^{(m)}_{23}, \overline{C}^{(m)}_{26}, \overline{C}^{(m)}_{33}, \overline{C}^{(m)}_{36}, \overline{C}^{(m)}_{44}, \overline{C}^{(m)}_{45}, \overline{C}^{(m)}_{55}, \overline{C}^{(m)}_{66}$ transformed material stiffnesses
$\overline{A}^{(\tau)}_{pq}, \tilde{A}^{(\tau)}_{pq}, A^{(\tau)}_{pq}$ elastic engineering stiffnesses
κ shear correction factor
$N_\varphi, N_s, N_{\varphi s}, N_{s\varphi}$ in-plane meridian, circumferential and shearing force resultants
$M_\varphi, M_s, M_{\varphi s}, M_{s\varphi}$ meridian, circumferential and torsional couple resultants
T_φ, T_s transverse shear force resultants
L_{pq} equilibrium operators
$q_\varphi, q_s, q_n, m_\varphi, m_s$ generalized external actions
$q^+_\varphi, q^+_s, q^+_n$ external forces in the three principal directions φ, s, ζ at the top surface of the shell
$q^-_\varphi, q^-_s, q^-_n$ external forces in the three principal directions φ, s, ζ at the bottom surface of the shell
N number of sampling points in φ direction
M number of sampling points in s direction
T number of sampling points in ζ direction
$\varsigma^{\varphi(1)}_{ik}$ GDQ weighting coefficients of the first order derivative in φ direction
$\varsigma^{s(1)}_{jk}$ GDQ weighting coefficients of the first order derivative in s direction
$\varsigma^{\zeta(1)}_{mk}$ GDQ weighting coefficients of the first order derivative in ζ direction
i generic discrete sampling point in φ direction
j generic discrete sampling point in s direction
m generic discrete sampling point in ζ direction
$\sigma_x, \sigma_y, \tau_{xy}, \tau_{xn}, \tau_{yn}, \sigma_n$ stress components
$\sigma_\varphi, \sigma_s, \tau_{\varphi s}, \tau_{\varphi n}, \tau_{sn}, \sigma_n$ stress components
$\varepsilon_\varphi, \varepsilon_s, \varepsilon_n, \gamma_{\varphi s}, \gamma_{\varphi n}, \gamma_{sn}$ strain components
q_0 uniformly distributed load
E Young modulus for the isotropic material
v Poisson coefficient for the isotropic material
(x,y) co-ordinate system of the rectangular plate
(x,s) co-ordinate system of the annular plate
R_i inner radius of the annular plate
R_e outer radius of the annular plate
$\boldsymbol{\delta}_b$ vector of the boundary degrees of freedom
$\boldsymbol{\delta}_d$ vector of the domain degrees of freedom
f_b external action vector of the boundary degrees of freedom
f_d external action vector of the domain degrees of freedom
$\mathbf{K}_{bb}, \mathbf{K}_{bd}, \mathbf{K}_{db}, \mathbf{K}_{dd}$ stiffness matrices of the boundary-domain partitioning
$\mathbf{t}_\varphi, \mathbf{t}_s, \mathbf{n}$ tangential and normal vectors related to the reference surface

comprehensive analysis for elastic isotropic shells and plates was made by Kraus [7] and Gould [14,15]. The present work is just based on the FSDT. In order to include the effect of the initial curvature in the evaluation of the stress resultants a generalization of the Classical Reissner–Mindlin Theory (CRMT) has been proposed in literature by Kraus [7], Qatu [16–18] and Toorani and Lakis [19]. There are three different ways to evaluate the engineering elastic constants. The first is the Classical Reissner–Mindlin approach [7] that consists in neglecting the effect of curvatures. Using this approach the engineering elastic stiffnesses are constant and do not depend on curvatures. The second, proposed by Kraus [7] and Toorani and Lakis [19], is based on the Taylor expansion, while the third proposed by Qatu [16–18] consists in the exact integration of the elastic constants. As a consequence of the use of these considerations the stress resultants directly depend on the geometry of the structure in terms of the curvature coefficients and the hypothesis of the symmetry of the in-plane shearing force resultants and the torsional couples declines. A further improvement of the previous theories of shells has been proposed by Toorani and Lakis [20]. In this work the kinematical model is generalized in order to include the effect of the curvature from the beginning of the shell formulation. By so doing the strains relationships have changed and, as consequence, the equilibrium equations in terms of displacements have to be modified. It is worth noting that no results are available in the literature about this theory for doubly-curved shells. In fact, some authors proposed good theories [16–19], but no considerations about doubly-curved shells of revolution have been done. Thus, the motivation of the present work is based on the lack of results about completely doubly-curved shells and panels of revolution. Furthermore, due to the significant developments that have taken place in composite materials [21], the increase in the their use in a lot of types of engineering structures in the last decades calls for improved analysis and design tools for these types of structures. Thus, in this paper, the laminated composite shells are considered.

As for the static analysis of shells, several studies have been presented earlier. With regard to semi-analytical methods, each static and kinematic variable is transformed into a theoretical infinite Fourier series of harmonic components [21–25]. However, applicability of the semi-analytical solutions are limited in terms of the boundary conditions, stacking sequences and the form of the panel. The finite element method do not present these drawbacks and it represents the most popular numerical tool in carrying out the above analyses [14,15,21,26–29]. Furthermore, meshless methods are also available to solve analogous problems such as reported in literature [30–34].

For a curved panel of general form it is not possible to perform a semi-analytical solution and the two-dimensional field must be dealt with directly, as will just be done in this paper. The