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Machine compliance and hardening effects on cavity growth in soft adhesives

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Abstract

The effect of surface tension on cavity nucleation and growth in an incompressible Neo-Hookean material under dead-loading has been studied previously (Dollhofer et al., *Int J Sol Struct*, 2004; 41: 6111–27). The above work is extended to include machine compliance and material hardening effects on cavity growth in soft adhesives in this paper. The equilibrium cavity stretch is found both by solving the field equations and by the energy approach. It is shown that the equilibrium cavity stretch is determined by three dimensionless parameters for the dead-loading case and four parameters for the finite compliance case for the Neo-Hookean material. For the Mooney–Rivlin material, one additional dimensionless parameter is needed.

Keywords: Pressure-sensitive; Tack; Hardening; Machine compliance

1. Introduction

Soft materials such as gels and lightly cross-linked elastomers have many applications in areas of technology such as pressure sensitive adhesives (PSA) and biological engineering. Since these materials are nearly incompressible, i.e. their shear moduli are orders of magnitude lower than their compressive moduli, failure often is initiated by the nucleation and growth of cavities, especially if the geometry is highly confined. Indeed, as a result of geometric confinement, the stress state is highly triaxial, favoring cavity nucleation and growth [1]. For example, in a probe tack test [2–5], a very thin layer of soft adhesive is bonded to a glass substrate. A steel punch, of radius much greater than the adhesive thickness, is brought into intimate contact with the adhesive. As the punch is removed, cavities are often observed to nucleate at the interface between the punch and the adhesive. If the interfacial adhesion is strong, these cavities grow into the adhesive layer, eventually evolving into a highly fibrillated structure formed on the cavity walls [2,3,6]. Similar tests have been employed to study the adhesion of barnacles to stiff substrates.

Probe tests were used first by Hammond [7] to measure the tack of adhesives, defined as the maximum tensile force during separation; then in the 1980s Zosel [8] argued that the integral under the stress–strain curve needed to be used to characterize tack. However it is only recently that the peak stress in the force-separation curve was attributed to the formation of cavities by Lakrout et al. [2]. Since this first result, Chiche et al. [9] and Roos [10] have shown that the peak stress cannot be easily related to the linear elastic modulus, G, of the soft adhesive as predicted by a simple cavitation model such as that first proposed by Gent [11]. In some cases, the peak stress depends on the layer thickness and in all cases it depends on the non-linear elastic properties of the material. These results suggest that boundary

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Nomenclature

λ	stretch
W	stored energy function
ω	restriction of W to a particular class of deformations
τ	Cauchy stress tensor
E	infinitesimal Young's modulus
c_{01}, c_{10}	material constants in the Mooney–Rivlin model
γ	surface energy per unit area
δ	applied displacement on rigid loading device
k	stiffness of springs
R, r	particle's distance from the origin in the reference and current configuration
A, B	cavity and exterior radii in the reference configuration
a, b	cavity and exterior radii in the current configuration
P^*, T^*	normalized nominal and true traction
α	parameter characterizing the role of surface tension, $\alpha = \gamma/EA$
β	parameter characterizing the role of machine compliance, $\beta = AE/k$

conditions are important in the growth of cavities in soft adhesives, and that other constitutive equations than simple Neo-Hookean should be considered. In this paper we try to address some of these questions by exploring models of single cavity growth.

There is a large volume of theoretical work on the deformation of spherical cavities in elastic solids. An excellent review of these works can be found in Horgan and Polignone [12]. The focus of these studies was to determine the equilibrium elastostatic fields and to study the dependence of the number of equilibrium solutions on the constitutive model. These past studies focused on traditional non-linear elastic solids such as vulcanized rubber, which is highly cross-linked and in which deformations due to surface tension can be neglected. Deformations due to surface tension cannot be neglected for elastic gels and pressure sensitive adhesives. Since these materials are only lightly cross-linked, their elastic moduli can be orders of magnitude lower than those of vulcanized rubbers, so that surface tension can play a significant role in their deformation. Also, highly crosslinked rubbers fail by crack propagation and models of cavity growth ahead of a crack-like cavity by breaking bonds have been studied by Williams and Schapery [13], Gent and Wang [14] and more recently by Lin and Hui [15]. Cavity growth by crack propagation is much less likely in soft materials like PSA, as the stress required to break bonds is much higher than the elastic modulus, even if hardening effects are taken into consideration.

Another important consideration in the modeling of cavity growth is the boundary condition. Most analyses assume that the cavitated body is loaded by an uniformly applied constant true traction field at infinity that is independent of the deformation of the cavity surface; see Green and Zerna [16], Gent and Tompkins [17], Ball [18]. This boundary condition is, for obvious reasons, difficult to achieve experimentally. In reality, both the finite size of a specimen and the loading machine compliances can play an important role in the deformation of a cavity. This work is a very slight step in this direction.

Gent and Tompkins [17] were the first to consider the effect of surface tension on cavity growth due to a remote applied pressure. For a spherical cavity in an infinite Neo-Hookean solid subjected to a remotely applied constant hydrostatic traction, the solution is found to be unique. However, this is not the case if one considers dead loading, where the traction changes with deformation in such a way so that the total applied force is independent of deformation. For this case, Dollhofer et al. [19] has recently shown that multiple solutions can exist. This was then used to explain the different growth rate of optically visible and invisible precursors and good agreement was obtained with probe tack experiments on soft adhesives.

The following questions naturally arise: is the existence of multiple solutions an artifact of the dead loading condition? In other words, can multiple solutions exist in loading configurations that take into account the effect of loading machine compliance? Also, how do the solutions depend on the constitutive model? In this paper, we extend the work of Dollhofer et al. [19] to address these issues. The approach here is also slightly different from the above-mentioned work, in which the equilibrium solutions were obtained from the potential energy landscape. In this work, exact solutions are obtained by solving the field equations. The physical significance of these solutions is discussed by examining the potential energy landscape.

The plan of this paper is as follows: in section two, we formulate the governing field equations used to obtain the equilibrium solution and then describe the equivalent energy approach. We then discuss the special cases of dead-loading and displacement-controlled tests in section three. Numerical results are given in section four for the Neo-Hookean model. The effect of material hardening is studied in section five by considering the Mooney–Rivlin material model, which is representative of the non-linear elastic properties of PSA; see Roos and Creton [20]. Comparison with previous experimental results, summary of the main results of this paper along with a discussion of future extensions of the present work is finally given in Section 6.