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Variational principle of partial-interaction composite beams using Timoshenko's beam theory

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ABSTRACT

Based on the kinematic assumptions of Timoshenko's beam theory, this paper formulates the principle of virtual work and reciprocal theorem of work for the partial-interaction composite beams. Then the principle of minimum potential energy and minimum complementary energy are derived and proved. The variational principles for the frequency of free vibration and critical load of buckling are also deduced afterward as well as the mixed variational principle with two types of variables. These variational formulae are all rendered in terms of shearing force, bending moment and axial force as well as corresponding deflection, rotation angle and interlayer slip, which can be applied conveniently for analyzing of composite beams. According to the proposed variational principles, the governing equations of static bending, free vibration and buckling can be obtained for the partial-interaction composite members as well as the corresponding boundary conditions. Finally, some numerical examples are presented and compared with the other solutions available in literatures to demonstrate the present theory.

1. Introduction

Due to their appropriate usage of high tensile strength of steel and high compressive strength of concrete, composite steel-concrete beams have widespread applications in engineering structures. Because of the finite rigidity of those connectors, which combined the steel and concrete parts to work together, longitudinal slips will occur at the interface between the two materials even under small load. This phenomenon is well known as partial-interaction [1].

The theory foundation of the partial-interaction composite beams is commonly attributed to Newmark et al. [1] who developed the linear relations between the interlayer shear force and slip and derived the governing equations of partial-interaction composite beams based on the classical beam theory. Goodman and Popkov [2] extended the linear relations to non-linear one for layered wood systems. Itani and Brito [3] presented the closed-form solutions of the second-order differential equation involving the axial force and interlayer slip derived by Newmark et al. [1]. Murakami [4] proposed the theory of partial-interaction composite members including the influence of shear deformation and derived the solution of a simply supported beam with a concentrated force at the mid-span. On the basis of classical beam theory, Girhammar and Gopu [5] presented the differential equation and its analytical solutions of partial-interaction composite members stressed by axial force and applied them to some boundary conditions. Recently, Wu et al. [6] analyzed the free vibrations of partial-interaction composite members with axial force. Consequently, they not only exhibited the exact formula of the free vibration of a simply supported beam, but also suggested approximate formulae of composite beams with other boundary conditions conveniently assumed in practical applications. Later on, Timoshenko's beam theory was used for detail investigation of the static, dynamic and buckling behavior of partial-interaction composite beams and some analytic solutions for different boundary conditions were obtained by Xu and Wu [7]. Schnabl et al. [8] also gave the analytical solutions of two-layer beam taking into account shear deformation, in which different shear deformations were allowed in two layers. Chen et al. [9] established a state space formulation for analyzing static response of partial-interaction composite members to deal with non-uniformly distributing shear connectors and continuous composite beams, and also explored the possibility of transverse separation at the interface. All the works above are based on the beam theories, Rao and Ghosh [10], together with Fazio and Hussein [11], however, analyzed the mechanical behaviors of partial-interaction composite members based on the theory of elasticity. Xu and Wu [12,13] also investigated partial-interaction composite beams using the assumptions of the plane stress. Regarding forced vibrations,

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Girhammar and Pan [14] studied the dynamic behavior of partial-interaction composite members presented the orthogonal relations of the mode shapes of vibration. Adam et al. [15] separated the dynamic responses into quasi-static and complementary dynamic responses for simplification of analysis. Grihammar et al. [16] reinvestigated the vibration equation and corresponding generalized boundary condition of partial-interaction composite members which considered the end with elastic support or lumped mass. More recently, Shen et al. [17] developed the orthogonality of mode shapes and analyzed the behaviors of forced vibrations of partial-interaction composite members by using symplectic inner product.

Numerical methods, especially finite element method (FEM), were also used widely in composite beams. For examples, Thompson et al. [18] developed a finite element of layered wood systems with interlayer slips. Itani and Hiremath [19] also studied the behavior of partial-interaction of composite beams by employing the finite difference method. Ayoub and Fillippou [20] presented the two-field mixed element for nonlinear steel–concrete composite beam to analyze the behaviors of the composite beams under cyclic load. Dall'Asta and Zona [21] derived three-field mixed formulation for the non-linear analysis of composite beams with deformable shear connection. Faella et al. [22] formulated the "exact" analytical expression of stiffness matrix of steel–concrete composite beams with flexible shear connection, and Ranzi et al. [23] then investigated the creep and shrinkage effects of concrete on the partial-interaction composite members. Čas et al. [24] developed a new finite element formulation for the non-linear analysis of two-layer composite planar frames with an interlayer slip based on the geometrically nonlinear Reissner's beam theory with small slip assumption. Ranzi and Bradford [25] developed an element for partial-interaction composite members to analyze the properties of continuous composite beams with multi-span in the cases of serviceability limit state and ultimate limit state. Ranzi and Zona [26] presented an analytical model for the analysis of steel–concrete composite beams with partial shear interaction, which is obtained by coupling an Euler–Bernoulli beam for the reinforced concrete slab to a Timoshenko beam for the steel beam. Sousa and DaSilva [27,28] presented an alternative procedure for nonlinear numerical analysis of composite beams, where the partial connection between the elements is dealt with especially designed interface elements. Schnabl et al. [29] proposed a locking-free two-layer Timoshenko beam element with interlayer slip. Zona and Ranzi [30] discussed three different beam models and relevant elements for non-linear analysis of composite beams with interlayer slip. Nguyen et al. [31] derived the exact stiffness matrix for a two-layer Timoshenko beam element with partial interaction.

Variational methods play an important role in the structural analysis since they can be used to derive the governing differential equations and develop approximate methods including FEM. Challamel and Girhammar [32], for example, investigated the buckling of partial composite beam-columns based on variational theories. Moreover, and perhaps most importantly, the variational method provides a natural means for approximation or establishes the foundation of the most powerful approximate methods. In the above-mentioned some works concerning FEM [18–31], the principles of virtual work and minimum potential energy were used to derive the formulation of the elements. However, these variational principles were not presented systematically and some variational principles were rendered in terms of stresses and strains rather than the stress resultants, deflection and rotation, which were more convenient to formulate based on beam theories. For this purpose, this paper presents the principle of virtual work and reciprocal theorem of work in terms of the stress resultants, deflection and rotation, and proves the principles of minimum potential energy and minimum complementary energy of partial-interaction composite beams. All the work is based on the kinematical assumptions of Timoshenko's beam theory, where the uniform shear deformation is assumed in two sub-elements. In addition, the variational formulae of frequency of free vibration and critical load of buckling are deduced, as well as the generalized variational principle with two types of variables. The analysis of buckling behavior is based on the Engesser theory [32], in which the axial force is chosen always parallel to the non-deformed beam axis. Finally, several examples are illustrated and demonstrated through numerical examples.

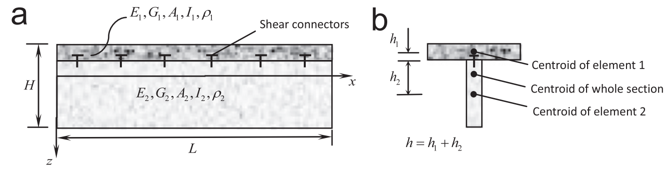

Fig. 1. Partial-interaction composite beams: (a) elevation and (b) cross section.

2. Formulation

2.1. Description of problems and assumptions

It is supposed a composite beam with two sub-elements of different materials in the *xz* plane, as shown in Fig. 1. The *x* axis is through the centroid of the whole cross-section of the composite beam in which E_i, G_i, I_i, A_i and $\rho_i (i=1,2)$ denote Young's modulus, shear modulus, moment of inertia, cross-sectional area and the mass density of the two sub-elements, respectively. L and H indicate the length and total height of the beam, respectively, and the symbols h_1 and h_2 are the distances from the centroids of the two sub-elements to the interface between the two sub-elements, respectively, and $h=h_1+h_2$.

In order to keep the integrity of this work, the assumptions for partial-interaction composite members in many literatures [7] are repeated as following: (1) all of the constitutive materials behave linearly and the deformations are small; (2) the shear connectors between the two sub-elements are continuous and distributed longitudinally[1] ; (3) the shear force of the shear connector is proportional to the interlayer slip; (4) no transverse separation occurs on the contact interface, which means that the curvature is the same for both sub-elements at any cross-section. (5) Timoshenko's beam theory is adopted for two sub-elements, i.e., transverse shear deformations of the cross-section are allowed together with the rotary inertia and the shear deformations are identical in the two sub-elements.

2.2. Kinematical relationship

In Fig. 2, u_1 and u_2 are the longitudinal displacements of the centroids of the cross sections of two sub-elements, respectively. The rotary angle is denoted by ψ, which is assumed to be equal in the two sub-elements. According to assumptions (4) and (5), the

[1] This assumption is somewhat different from others available in literatures where the shear connectors are assumed uniformly distributed. In practical, the shear connectors are usually non-uniformly distributing, even discrete. This case cannot be dealt conveniently in a strong form, i.e., differential equations, of governing equations of composite beams. On the contrary, it is natural in the weak form. Thus, the assumption of uniformly distribution of shear connector is dropped. Furthermore, the present energy methods can readily cope with the discrete shear connectors although the assumption of continuously distributing of shear connectors is remained for the sake of simplification of formulation.