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An airy function to rapidly predict stresses in wound metal strip having asymmetric thickness profile

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ABSTRACT

With increased demand for thin gage flat metals, control of strip flatness or shape in cold rolling processes has become very important. To improve the flatness quality of cold rolled metal strip and sheet, this work provides a rapid method to predict the transient strains (or stresses) occurring during the rewinding of flat-rolled steels having problematic asymmetric strip thickness profile (or wedge). Flatness control systems, used to monitor and correct the distribution of stress across the width of rolled sheet, are unable to distinguish between stresses induced during rolling, and those caused when rewinding strip containing asymmetric thickness profile. The winding stresses, unless large enough to plastically deform the strip, vanish upon unwinding during subsequent operations such as stamping. Therefore, to help avoid strip flatness defects in thin strip containing wedge, a method is developed to separate the winding stress contribution from the overall stresses that are measured indirectly by flatness control systems. A fourth-order polynomial Airy function is developed to rapidly predict the in-plane stresses based on mandrel wrap number and spatial location on the strip. The Airy function is obtained by applying two-dimensional finite element analysis to study the transient in-plane stresses during rewinding at various numbers of mandrel wraps for a strip containing wedge profile. Three-dimensional finite element analysis is first employed, however, to show justification to a simplified two-dimensional problem described by the plane-stress Airy function. The two-dimensional finite element analysis provides insight as to how the in-plane stresses evolve, and allows determination of coefficients for the Airy function based upon model geometry and displacement boundary conditions. This approach differs from other methods that employ Fourier series to solve the biharmonic equations for an assumed two-dimensional problem. Finally, filtering of the winding stresses from flatness control system input signals is also discussed based on data taken from a rolling mill different to that used for model development.

1. Introduction

1.1. Strip flatness

As products derived from metal sheet become smaller and more precise, the demand for higher quality raw materials also increases. When cold rolling thin metal strip or sheet containing asymmetric thickness profile, quality problems such as poor final flatness are difficult to address. This is particularly true because the rolling and winding of strip with asymmetric thickness profile can cause localized buckling (strain defects) due to high in-plane stress gradients. The resulting poor strip flatness also decreases rolling productivity, and may render the products unusable in downstream manufacturing processes such as annealing, slitting, or stamping. An overview of the cold rolling and subsequent winding process for strip with less-problematic, symmetric thickness profile has been discussed by Edwards and Boulton [1]. In contrast, the focus of this work is an analytical method to rapidly assess in-plane stress (or strain) and related flatness for strips with asymmetric thickness profile (or wedge). Parameters for the study are based on the reversing cold rolling mill operated by Mid America Stainless Corp (Fig. 1a and b).

Almost all of the type 301 and 304 stainless steels rolled on the narrow Mid America Stainless mill contain up to three percent wedge [2]. This is because the narrow coils rolled are obtained from wider coils containing up to 3% parabolic thickness profile. Parabolic and wedge-type strip thickness profiles, corresponding to those before and after slitting, are depicted in Fig. 2.

1.2. Flatness implications when winding strip with asymmetric thickness profile

Flatness or shape defects in rolling occur mostly where localized regions of the strip contain excess longitudinal strain (elongation in the rolling direction) as compared to surrounding

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Nomenclature

N	number of wraps (or laps)
C_1^N	circumference of winding at thicker edge of strip
C_2^N	circumference of winding at thinner edge of strip
D	diameter of mandrel
R	radius of mandrel
D_1^N	diameter of winding at thicker edge of strip
D_2^N	diameter of winding at thinner edge of strip
T	thickness at thicker edge of strip
t	thickness at thinner edge of strip
W	width of strip
L^N	length of strip for N mandrel wraps
δ^N	difference in distance traveled at wrap N (equivalent to $C_1^N - C_2^N$)
E	elastic modulus of strip
G	modulus of rigidity of strip
Φ	Airy function
ε_x	normal strain in the rolling direction
ε_y	normal strain in the widthwise direction
γ_{xy}	in-plane shear strain
σ_x	normal stress in rolling direction
σ_y	normal stress in widthwise direction
σ_z	normal stress in out-of-plane direction
τ_{xy}	in-plane shear stress

regions. Upon removal of externally applied tension, the excess strain manifests itself in the form of center-buckles, edge waves, or more complex flatness defects. An especially problematic condition in which buckling occurs, and the motivation for this work, is due to the mis-correction of flatness control systems when rolling and winding strip with wedge-type thickness profile. Wedge of only 2% or 3% of nominal strip thickness can present a major challenge in the stability and quality control of cold rolling operations, due to resulting flatness defects.

Flatness control systems monitor and correct strip flatness defects that arise from the non-uniform distribution of in-plane stress across the strip width. The systems operate by detecting the radial force applied by the strip as it passes at an angle over a sensor roll, just after exiting the mill. As illustrated in Fig. 3, flatness corrections are frequently made by bending, shifting, or crossing the rolls to adjust the distribution of rolling force and the resulting in-plane stress field.

The sensor roll in a flatness control system is typically composed of a series of piezoelectric or other sensors located along the axis of the roll. Each sensor records the magnitude of radial force applied by a portion of the strip as it passes over the sensor roll prior to being wound on a mandrel. By simply measuring the magnitude of radial force, however, the flatness sensors are unable to distinguish between non-uniform radial forces caused by deformation stresses due to the rolling, and those induced by the mandrel winding operation. The deformation stresses occur at the work rolls, and result from differences in local plastic deformation across the strip width. The stresses due to winding are usually elastic, and vanish accordingly when the strip is unwound and free of the applied tension used to facilitate rolling.

When winding a strip containing wedge, the thickness variation causes stress gradients across the strip width that are transient, and therefore based on mandrel wrap number. This results from the additional distance traveled by thicker regions of the strip during winding. As the number of mandrel wraps grows, the difference between the maximum and minimum diameters of the wound strip (corresponding to thickest and thinnest edges) becomes significant. If the relative wedge is large enough, the sensor will begin to detect and correct for a force distribution dominated by the winding stress gradient rather than the intended rolling stress gradient. Hence, by not being able to distinguish winding stresses from deformation stresses, the flatness control system is unable to make the correct adjustments to the mill's flatness control devices. When the strip is later unwound (free of applied tension) for stamping or slitting operations, the *mis-corrected* flatness defects are often clearly visible. The presented work addresses the winding stress problem

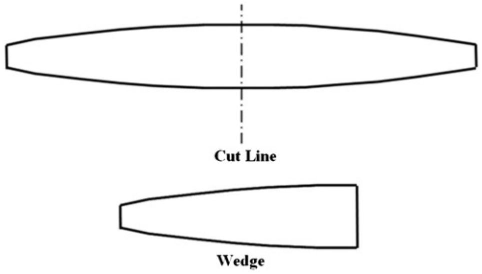

Fig. 2. Wedge-type thickness profile, obtained after slitting wider strip with parabolic profile.

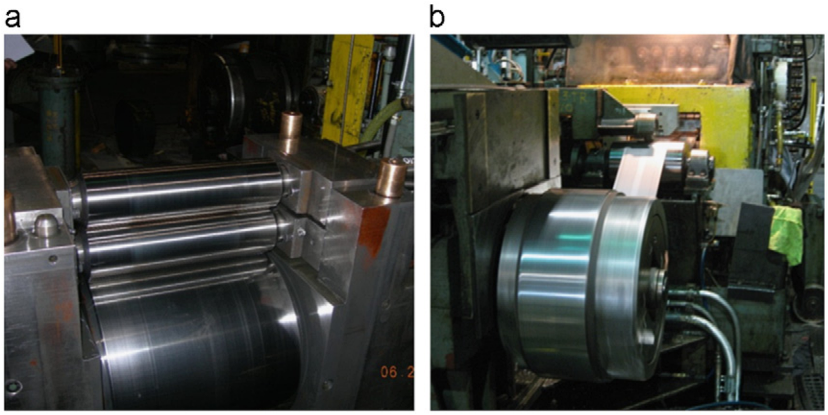

Fig. 1. (a) Mid America Stainless Corp. 255 mm 4-high mill (upper backup roll not shown). (b) Mandrel winder on Mid America Stainless 4-high mill.