International Journal of Mechanical Sciences 53 (2011) 485–493

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Nonlinear study of large deflection of simply supported piezoelectric layered-plate under initial tension

Chun-Fu Chen*, John-Han Chen

Department of Mechanical Engineering, Chung-Hua University, No. 707, Section 2, Wu-Fu Road, Hsin Chu, Taiwan 30067, ROC

ARTICLE INFO

Article history:
Received 28 November 2009
Received in revised form
23 January 2011
Accepted 3 April 2011
Available online 17 May 2011

Keywords:
Large deflection
Piezoelectric layered plate
Initial tension
von Karman plate theory

ABSTRACT

The nonlinear problem of large deflection of a simply supported piezoelectric layered plate under initial tension is studied. The approach follows von Karman's plate theory for large deflection for a symmetrically layered isotropic case including a piezoelectric layer. The nonlinear governing equations are solved using a finite difference method, by taking the associated linear analytical solution as an initial guess in the numerical iteration procedure. The results for a nearly monolithic plate under a very low applied voltage are found to correlate well with available solutions for a single-layered case under pure mechanical loading and thus the present approach is validated. For three-layered plates made of typical silicon based materials, various initial tension and lateral pressure are considered, and different applied voltages up to a moderate magnitude are implemented. No edge effect was observed, in contrast to the cases of clamped plates in literature. In additions, varying the layer moduli seems to have an insignificant effect upon the structural responses of the layered plate. On the other hand, the piezoelectric effect tends to be apparent only in a low pretension condition. For a relatively large pretension, the effect of initial tension becomes dominant, yielding nearly unique solutions for the structural responses, regardless of the magnitudes of the applied voltage and the lateral pressure.

1. Introduction

A pressure-sensing device may often be operated under the condition of large deflection. Classical plate or lamination theory following the Kirchhoff assumption is no longer applicable in this case and hence a more advanced theoretical approach must be considered to evaluate their structural behaviors. In literature, there have been quite a few studies available that dealt with this type of problem. The earliest work may trace back to the approach discussed in the well-known textbook of Timoshenko and Woinowsky-Krieger [1]. Yet, it appeared to be due to Voorthuyzen and Bergveld [2], first, to solve for the problem of large deflection of a uniformly loaded circular diaphragm simulating a micro-sensing device under initial in-plane loading by the numerical method of finite difference. They included both bending and tensile stresses in formulating the problem of large deflection but only the results of deflection at the center of the plate were presented, including the study of effect of tensile force. A rigorous study of nearly the same problem was conducted by Sheplak and Dugundji [3] that presented both linear analytical solution and nonlinear numerical results. The influences of initial tension and lateral load upon the geometrical responses were illustrated and the behavior of a plate transforming from a plate mode to a membrane was thoroughly investigated. Based on a similar approach, the problem of large deflection of an annular plate with a rigid boss was later considered by Su et al. [4].

Following the previous works, the merit of a linear study was clearly illustrated that it may provide not only the primary insight for the problem but also a valuable initial prediction for solving the original nonlinear problem in a numerical procedure. Nevertheless, available studies thus far seem to have discussed mainly clamped-ended conditions and the other supporting cases i. e., the simply supported problems appear to have received relatively less attention in literature. One problem of simple support was examined by Lin et al. [5] and a simulation program was developed for investigating the sensitivity and behavior of linearity of a piezoresistive sensor, but only classical plate theory was utilized. A recent work considering geometrically nonlinear response of a circular plate under piezoelectric actuation was presented by Kapuria and Dumir [6], but only moderate large deflection was taken into account, neglecting the direct piezoelectric effect in problem formulation. In fact, although it may be close to a completely clamped situation, the edge around a typical micro-machined flat element is not completely clamped in reality. Solutions for simply supported problems of this type are thus desirable as they may provide an opposite bound for the structural responses.

In addition to large deflection, a micro-sensing device is very often subjected to initial tension. The cause may be due to the

* Corresponding author.
E-mail address: cfchen@chu.edu.tw (C.-F. Chen).

0020-7403/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijmecsci.2011.04.003

Nomenclature

N_0	initial tensional load
p_0	uniform lateral pressure
r	radius of the plate
h_i	thickness of the typical *i*th layer of the plate
h	total thickness of the layered plate
E_i	Young's modulus of the typical *i*th layer of the plate
ν_i	Poisson's ratio of the typical *i*th layer of the plate
i	index of a typical layer in the plate
u	radial displacement of the plate
w	lateral deflection of the plate
(N_r, N_θ)	(radial, circumferential) force resultant of midplane
Q_r	transverse shear force resultant
A_l, A_t	elements of extensional stiffness matrix of the plate
ε_r, ε_θ	(radial, circumferential) strain of the plate
M_r, M_θ	(radial, circumferential) moment resultant
κ_r, κ_θ	(radial, circumferential) curvature
D_l, D_t	elements of bending stiffness matrix of the plate
N_r^P, N_θ^P	(radial, circumferential) piezoelectric force resultant
M_r^P, M_θ^P	(radial, circumferential) piezoelectric moment resultant
$\hat{N}_r$, $\hat{N}_\theta$	(radial, circumferential) incremental force resultant
S_r, S_θ	non-dimensional (radial, circumferential) force resultant
$[\boldsymbol{q}]$	reduced stiffness matrix of typical *i*th layer of the plate
P	non-dimensional lateral pressure
k	non-dimensional initial tension parameter
ξ	non-dimensional radial coordinate
W	non-dimensional lateral deflection
U	non-dimensional radial displacement
θ	non-dimensional lateral slope
Ψ	non-dimensional curvature
V	applied voltage
d_{31}	piezoelectric constant of the piezoelectric layer
E_f	electric field constant
$\Delta\xi$	normalized radial division interval

micro-fabrication or micro-machining process for a miniaturized pressure sensor made of silicon-based semi-conductor materials. Significant pretension that may result in a drastic degradation in structural performance such as the deflection-based pressure sensitivity has been discussed by Cho et al. [7] and Chau and Wise [8]. A drastic variation of structural and physical responses near the edge was observed for the clamped-edged problems. The size of such edge region was mathematically defined [3], but only for single-layered isotropic cases. The linear analytical approach presented by Sheplak and Dugundji [3] was later utilized by Saini et al. [9] in studying the scaling relations of piezo-resistive microphones in which mechanical sensitivity of a sensing plate is analytically formulated and the effect due to initial tension was investigated. Apparently, the question regarding possible edge effect and the response of mechanical sensitivity for a simply supported case can be worth note for both design and application concerns.

This study was thus motivated to extend the work of Sheplak and Dugundji [3] to a simply supported symmetrically layered case including a piezoelectric layer. von Karman's plate theory for large deflection was employed in deriving the nonlinear governing equations including the consideration of piezoelectric effect. These equations were further expressed in a non-dimensional form, in terms of lateral slope and radial force resultants. The simplified linear problem was lately investigated by the present authors [10] in which an analytical solution expressible in terms of modified Bessel's functions was presented. The present study re-examined the original nonlinear problem by employing a numerical approach of finite difference method. An iteration scheme was established by taking the corresponding linear solution as an initial guess. The nonlinear geometrical responses including the central and overall lateral deflections, lateral slopes and curvatures along the radial direction of the plate were obtained. The influence due to the variations of initial tension, lateral load and the applied voltage across the piezoelectric layer upon various geometrical responses will be discussed, including the transition behavior for the plate to become a membrane. The distinction between the present nonlinear solutions and the previous linear solution will be illustrated as well.

2. Formulation of the problem and solution

A simply supported circular layered plate embedded with a piezoelectric layer polarized in the *z*-direction is considered. It is subjected to an initial in-plane tension, N_0, a uniform lateral load, $p_z = p_0$, and an applied voltage V, across the piezoelectric layer of thickness h_P as shown in Fig. 1. The governing equations based on force and moment equilibrium can be formulated in terms of lateral slope, w_r, in-plane force resultants, N_r and N_θ, and the moment resultants, M_r and M_θ, to take the following form:

$$(rN_r)_{,r} - N_\theta = 0, \quad (rQ_r)_{,r} + (rN_r w_{,r})_{,r} = -p_0 r, \quad (rM_r)_{,r} - M_\theta - rQ_r = 0, \tag{1}$$

where Q_r is the transverse shear force resultant, and Ns and Ms are the force resultants and moment resultants, respectively, defined through the laminate constitutive laws, i. e.,

$$N_r = A_l\varepsilon_r^\circ + A_t\varepsilon_\theta^\circ, \quad N_\theta = A_t\varepsilon_r^\circ + A_l\varepsilon_\theta^\circ;$$
$$M_r = D_l\kappa_r + D_t\kappa_\theta, \quad M_\theta = D_t\kappa_r + D_l\kappa_\theta. \tag{2}$$

In the above, a subscript r (θ) denotes a radial (circumferential) component, As and Ds are the elements of the extensional and bending stiffness matrix, respectively, expressible in terms of layer moduli and thickness of the plate; and the subscript l (t) represents the diagonal (off-diagonal) component, i. e.,

$$A_l = \sum_{i=1}^{n} \frac{E_i h_i}{1-\nu_i^2}, \quad A_t = -\sum_{i=1}^{n} \frac{\nu_i E_i h_i}{1-\nu_i^2};$$
$$D_l = \frac{1}{3}\sum_{i=1}^{n} \frac{E_i}{1-\nu_i^2}(z_i^3 - z_{i-1}^3), \quad D_t = \frac{1}{3}\sum_{i=1}^{n} \frac{\nu_i E_i}{1-\nu_i^2}(z_i^3 - z_{i-1}^3).$$

In addition, ε_α° and κ_α ($\alpha = r, \theta$) are the mid-plane strain and curvature components defined by the radial displacement, u, and the lateral deflection, w, such that,

$$\varepsilon_r^\circ = u_{,r} + (w_{,r})^2/2, \quad \varepsilon_\theta^\circ = u/r; \quad \kappa_r = -w_{,rr}, \quad \kappa_\theta = -w_{,r}/r. \tag{3}$$

Here, the nonlinear term, $(w_{,r})^2$, in the expression of radial strain, ε_r°, is what the governing equations will be different from the classical plate theory.

3. Nonlinear governing equations

The nonlinear governing equations can be derived in a way similar to those derived by Sheplak and Dugundji [3]. Thus, Q_r in the second equation of Eq. (1) is replaced by the moment resultants through the third and first. Following Eq. (2), then, the second of Eq. (1) can be manipulated to give an equation in