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Free vibration analysis of rectangular plates with internal columns and uniform elastic edge supports by pb-2 Ritz method

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ABSTRACT

Free vibration analysis of rectangular plates with internal columns and elastic edge supports is presented using the powerful pb-2 Ritz method. Reddy's third order shear deformation plate theory is employed. The versatile pb-2 Ritz functions defined by the product of a two-dimensional polynomial and a basic function are taken as the admissible functions. Substituting these displacement functions into the energy functional and minimizing the total energy by differentiation, leads to a typical eigenvalue problem, which is solved by a standard eigenvalue solver. Stiffness and mass matrices are numerically integrated over the plate using the Gaussian quadrature. The accuracy and efficiency of the proposed method are demonstrated through several numerical examples by comparison and convergency studies. Many numerical results for reasonable natural frequency parameters of rectangular plates with different combinations of elastic boundary conditions and column supports at any locations are presented, which can be used as a benchmark for future studies in this area.

1. Introduction

Rectangular plates with inner column supports and edge elastic supports are extensively used in mechanical, civil, ships and aircraft manufacturing. A good understanding of the dynamic behaviors of these structural components is crucial to the design and performance evaluation of mechanical systems. With its potential applications, the vibration characteristics of such kinds of plates have received considerable attention from researchers.

A vast body of literature for free vibration studies of plates is available. Excellent reviews have been made by Leissa [1–3] and Liew et al. [4]. Most of the previous studies, however, have been confined to plates having unique simple plan-forms with simple uniform boundary conditions. It is well known that the classical boundary conditions such as clamped, simply supported and free are relatively easy to formulate, but difficult to apply in practice. However, the boundary conditions may have significant influences to the free vibration characteristics of rectangular plates. Therefore, it is necessary to establish new models to evaluate the dynamic behaviors accurately. Considerable researchers use elastic springs to model the plate boundary conditions, and numerous papers have been presented to study the free vibration of plates using this model. Gorman [5,6] studied the free vibration of rectangular plates with elastic edge supports based on the classical plate theory and the Mindlin plate theory using the superposition method. Xiang et al. [7] used polynomials and basic functions as the admissible functions to investigate the free vibration of rectangular Mindlin plates with elastic boundary conditions by the Ritz method. Saha et al. [8] used the vibrating Timoshenko beam functions, and Zhou [9] applied the static Timoshenko beam functions, as the admissible function to analyze the same problem by the Rayleigh–Ritz method. In the seventies of the last century Laura et al. [10,11] presented the pioneer work on the free and forced vibration of circular plates having flexible supports. Recently, Laura and Avalos [12] investigated the transverse vibration problem of circular plates with an eccentric rectangular cutout elastically restrained against rotation and translation on both edges. The Raleigh–Ritz method was employed to obtain the first four frequency coefficients.

About the internal column support, there are several models to describe the effect of a column on the vibration of a plate. The simplest model is to take the column as a rigid point support, which takes the stiffness of the column as infinite in axial direction. Gorman [13,14] presented the solutions for rectangular plates with point supports, based on thin plate theory and the Mindlin plate theory by the superposition method. Liew et al. [15] investigated the Mindlin plates of arbitrary shapes with internal point supports by the Rayleigh–Ritz method. Kim and Dickinson [16] used the Lagrangian multiplier method combined with the orthogonally generated polynomials to study the rectangular plates with point supports. Zhou and Cheung [17,18] studied the free vibrations of tapered rectangular plates and composite

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Nomenclature

x,y,z	coordinate variables
ξ,η	non-dimensional coordinate variables
a	length of the plate in x direction
b	width of the plate in y direction
h	thickness of the plate
H	height of the column
$\tilde{u}$,$\tilde{v}$,$\tilde{w}$	displacements at a position in x,y and z directions
u,v,w	displacements at a position in the middle plane of the plate in x,y and z directions
θ_x	rotation of the plate normal about x-axis
θ_y	rotation of the plate normal about y-axis
Π	functional of the plate–column system
U_{max}	maximum potential energy of the plate–column system
U^P	maximum potential energy of the plate
U^S	maximum potential energy of the springs uniformly distributed at four edges of the plate
U^C	maximum potential energy of the columns in the plate domain
E	elastic modulus
G	shearing modulus
ν	Poisson's ratio
ρ	density of the plate per unit volume
D	flexural rigidity of a plate
k_L	stiffness of the lateral spring
k_R	stiffness of the rotational spring
k_T	stiffness of the torsional spring
μ_L	coefficient between the stiffness of lateral spring and the bending stiffness of the plate
μ_R	coefficient between the stiffness of rotational spring and the bending stiffness of the plate
μ_T	coefficient between the stiffness of torsional spring and the bending stiffness of the plate
k_f	lateral stiffness of the column
k_x	rotational stiffness of the column in x direction
k_y	rotational stiffness of the column in y direction
I_x,I_y	second moments of area of the column about x and y axes
i,r,q	index numbers
p_i	degrees of the two-dimensional polynomial space
c_i,d_i,e_i	unknown coefficients to be determined
c,d,e	sub-vectors of the unknown coefficients
q	vector of unknown coefficients consist of c,d,e
m,n,l	dimensions of the vectors c,d,e
ϕ_i,ψ_{xi},ψ_{yi}	pb-2 Ritz function consist of a monomial and a basis function
Φ,Ψ_x,Ψ_y	pb-2 Ritz function vectors
B,C,D	basic functions consist of the product of each boundary equation
K	stiffness matrix
M	mass matrix
ω	circular frequency
λ	non-dimensional frequency parameter
δ_i	governing factors
S_i	governing factors

pates with point supports, respectively, using the static beam functions as the displacement functions. Petyt and Mirza [19] carried out the finite element analysis for the free vibrations of column supported floor slabs by simplifying the columns as pinned point supports. Narita [20–22] presented a series solution for the free vibration analysis of rectangular plates with complex mixed conditions, point supports and cantilever plates with point constraints. Bapat et al. [23–25] discussed the vibration characteristics of rectangular plates having various types of supports such as a single point support, arbitrary multiple point supports within the plate and at the edges. Numerous papers have been reported on the study of free vibration problems of point supported plates; however, this model cannot generate reasonable results for column support plates because the stiffness of the columns has not been considered. An improved model is to take the column as three massless springs in the longitudinal and two rotational directions, which considers the effect of the stiffness of the column. Lee and Lee [26] investigated rectangular plates on elastic point supports and discussed the effects of the support stiffness. Huang and Thambiratnam [27] studied rectangular plates on elastic intermediate supports using the finite strip element method combined with a spring system. Zhou and Ji [28] presented a direct exact solution for the free vibration of thin rectangular plates with two opposite edges simply supported and with internal column supports. The effects of column sizes and column models on the natural frequencies of a plate–column system have been discussed in detail.

The previous publications, however, have concentrated their studies on the case of either elastic edge support or inner supports alone, and there is little reported research for the free vibration analysis of plates with simultaneous elastic edge and internal supports. Ohya et al. [29] are the pioneers to analyze the vibration problem of plates with both elastic edge supports and internal column supports. They obtained an analytical solution for this problem via the superposition technique based on the first order shear deformation theory. A great deal of new numerical results is reported. However, only symmetric column distribution has been considered in their study, where the vibration modes are either symmetric or anti-symmetric. If the columns distribute in arbitrary location, this method may lose its suitability and applicability. This is the motivation for this paper, the aim of which is to propose a general method to study the free vibration problem of plates with elastic edge supports and arbitrarily located column supports.

In this paper, the powerful solution technique, pb-2 Ritz method combined with Reddy's higher order shear deformation plate theory are employed to treat the free vibration analysis of rectangular plates on elastic edge supports and internal column supports. Two models of column are examined in this paper, one is taking it as three elastic massless springs in longitudinal and two rotational directions; the other is taking it as local uniformly distributed elastic springs only in the longitudinal direction. The correctness and numerical accuracy of the present method is first verified by the comparison of the present results with corresponding exact solutions or other numerical solutions in the open literature. Then parametric studies are conducted to examine the effects of the locations, sizes of the columns and different column models as well as the edge elastic support stiffness on the dynamic behavior of the plate–column system.

2. Theoretical analysis

2.1. Displacement field

Consider a rectangular plate of length a, width b and thickness h with several internal columns as shown in Fig. 1. It is assumed