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Characterisation of crack tip stresses in elastic-perfectly plastic material under mode-I loading

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Asymptotic crack tip stress fields are developed for a stationary plane strain crack in incompressible elastic-perfectly plastic material under mode-I loading. Detailed investigations have revealed that in between the two extreme conditions of crack tip constraint, that is, between the fully plastic Prandtl [1] field and the uniform stress field the most general elastic–plastic crack tip fields can be completely described by the 5-sector stress solution proposed in this article. The 3-sector stress field proposed by Li and Hancock [2] and the 4-sector field proposed by Zhu and Chao [3] are subsets of the general elastic–plastic field proposed in this work. This study has revealed that cases arise where the severe loss of crack tip constraint can lead to compressive yielding of crack flank. This particular situation leads to 5-sector stress field. Detailed studies have revealed that, in the most general case of elastic–plastic crack tip fields, the T_π parameter proposed by Zhu and Chao [3] cannot be used as a constraint parameter to represent a unique state of stress at the crack tip. A new constraint-indexing parameter $T_{CS\text{-}2}$ is proposed, which along with T_p is capable of representing the entire elastic–plastic crack tip stress fields over all angles around a crack tip. Excellent agreement is obtained between the proposed asymptotic crack tip stress field and the full-field finite element results for constraint levels ranging from high to low. It is demonstrated that the proposed constraint parameters are adequate to represent the crack tip constraint arising due to specimen geometry and loading conditions as well as the additional constraint that arises due to weld strength mismatch.

1. Introduction

Characterisation of crack tip stresses has been an area of active research for many decades. Williams [4] in his landmark paper showed that the crack tip stress fields in an isotropic elastic material can be expressed as an infinite series where the leading term exhibit a $1/\sqrt{r}$ singularity and the second term is independent of r. Classical fracture mechanics theory neglects all but the singular term and, thus, came the concept of characterisation of crack tip stresses by a single parameter. Although the third and higher order terms of Williams's series vanish near the crack tip, the second term (that is constant) remains finite and has a strong effect on the stresses in the plastic zone. This second term has been referred in the literature as T-stress. The single parameter characterisation is rigorously correct only for $T>0$. It is important to note that T-stress is an elastic parameter and has no physical meaning under large-scale plasticity. Then, assuming small-strain formulation, Hutchinson [5] and Rice and Rosengren [6] proposed the dominant term of the singularity field (often referred as HRR solution) for plane strain mode-I crack based on the J-integral [7]. Thus, the HRR singularity is the natural extension of one-parameter characterisation concept to a non-linear elastic material. However, it has been realized that the specimen geometry and loading conditions significantly affect the crack tip fields and, thus, the HRR field has limited application to real cracked structures. For a Ramberg–Osgood material model, the crack tip fields in the plastic zone can be expressed in terms of a power series where the HRR solution is the leading term. The higher order terms of this power series were grouped together and its amplitude was denoted as Q by O'Dowd and Shih [8]. Other representative two parameters that are used to characterise the crack tip stress fields are J–T of Betegon and Hancock [9] and J–A_2 of Chao et al. [10].

For a rigid plastic material (non-hardening), slipline fields (SLF) have been extensively used to estimate crack tip stresses in fully plastic state under plane strain condition. Results indicate that for a non-hardening material, under fully yielded condition, the stresses near the crack tip are not unique but a strong function of specimen geometry and loading condition. An excellent compilation of various SLF solutions has been given by McClintock [11]. For high constraint geometries like deeply cracked Double Edge Crack Plate in tension (DECP) plasticity completely surrounds the crack tip (Prandtl field) and SLF analysis can be used to obtain crack tip stress distribution

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over all angles. However, when constraint at the crack tip is not high enough to cause tensile yielding of the crack flank then an elastic sector appears there (not in all cases as would be discussed later) and SLF analysis can only describe the state of stress in the plastic sectors ahead of the crack tip.

Apart from analytical studies (SLF analysis) detailed numerical (finite element) investigations have been performed to evaluate crack tip stress fields in non-hardening material. For small-scale yielding, Du and Hancock [12] examined the effects of elastic *T*-stress on crack tip stress fields for mode-I crack under plane strain condition. They provided an explanation to the existence of incomplete crack tip plasticity in terms of *T*-stress. Under large-scale yielding Lee and Parks [13] analysed single edge cracked specimen subjected to combined bending and tension. Kim et al. [14] and Zhu and Chao [15] performed detailed FE analyses of SENB, CCP and DECP specimens. Further comments can be seen in Ref. [3]. It is important to understand that the two parameters such as J–Q or J–A_2 characterisation cannot describe the state of stress in the elastic sector near the crack flanks.

To construct the general elastic–plastic crack tip stress field Ibrgimov and Tarasyuk [16] and Nemat-Nasser and Obata [17] first discussed the possibility of existence of elastic sector for plane strain mode-I crack in elastic-perfectly plastic material. Then, Li and Hancock [2] described the crack tip fields under small-scale yielding in terms of plastic sectors and an elastic sector to account for the incomplete crack tip plasticity observed from detailed FE investigations. More details about elastic–plastic crack tip stress fields under mode-II and mixed-mode loading can be found in Ref. [3]. In the asymptotic solution, Li and Hancock [2] assumed three different stress sectors near the crack tip, that is, a constant stress sector (plastic), a fan field (plastic) followed by an elastic stress sector near the crack flank. Their 3-sector solution was extended by Zhu and Chao [3] who, based on available FE results of Kim et al. [14] and Zhu and Chao [15], proposed that the actual stress field of a stationary crack in elastic-perfectly plastic material under plane strain condition can be described by a 4-sector solution. Closed-form asymptotic solutions of crack tip fields were developed by them. Two undetermined parameters T_p and T_π were proposed to characterise the state of stress near the crack tip. The proposed asymptotic solutions were compared with detailed FE results for various fracture specimens with constraint level ranging from high to low.

In this article asymptotic crack tip stress fields are developed for a stationary plane strain crack under mode-I loading. Incompressible, elastic-perfectly plastic material with Von-Mises yield criterion was assumed for the present study. Detailed investigations have revealed that in between the two extreme conditions of crack tip constraint, that is, between the fully plastic Prandtl field and the uniform stress field the most general elastic–plastic crack tip fields can be completely described by the 5-sector stress solution. The 4-sector field proposed by Zhu and Chao [3] is a subset of the general elastic–plastic field proposed in this work. It is well known that loss of constraint at the crack tip leads to an elastic sector at the crack flank, thus, leading to incomplete crack tip plasticity. This study has revealed that cases arise where severe loss of crack tip constraint can lead to compressive yielding of crack flank. This particular situation leads to 5-sector stress field where the elastic sector is sandwiched between the two plastic sectors of uniform stress state. Such 5-sector stress field exists in an overmatched weld where the relatively higher strength of weld leads to shielding effect on the crack tip and, thus, leads to loss of crack tip constraint. Detailed 2D elastic–plastic finite element analyses were performed on mismatch welded centre crack panel (CCP) to examine validity of the proposed 5-sector stress field. Both undermatched and overmatched cases were analysed to simulate constraint levels ranging from high to low. Excellent agreement is obtained between proposed asymptotic crack tip stress field and finite element results. Detailed studies have revealed that, in the most general case of elastic–plastic crack tip fields, the T_π parameter proposed by Zhu and Chao [3] cannot be used as a constraint parameter to represent a unique state of stress at the crack tip. A new constraint-indexing parameter $T_{CS\text{-}2}$ is proposed, which along with T_p parameter, suggested by Zhu and Chao [3], is capable of representing the entire elastic–plastic crack tip stress fields over all angles around a crack tip. Advantages of the proposed $T_{CS\text{-}2}$ parameter over the T_π parameter are discussed. It is demonstrated that the proposed constraint parameters are adequate to represent the crack tip constraint arising due to specimen geometry and loading conditions as well as additional constraint that arises due to weld strength mismatch.

2. Governing equations

We consider here a stationary crack in an incompressible elastic-perfectly plastic material under plane strain condition. Zhu and Chao [3] have concluded that constraint has no effect on the stress state ahead of the crack tip for a mode-II crack in an elastic-perfectly plastic material. A similar conclusion was made by Chao and Yang [18] for a power-law hardening material. In view of the above-mentioned conclusions, only mode-I loading is considered here.

2.1. Equilibrium equations

For elastic-perfectly plastic material numerical results of Dong and Pan [19] have established that all stress components near the crack tip are bounded and are, thus, functions of polar angle θ only. The equilibrium equations, thus, reduce to ordinary differential equations and can be expressed in polar co-ordinate system, centred at the crack tip, in the following form:

$$\frac{d\sigma_{r\theta}}{d\theta}+\sigma_{rr}-\sigma_{\theta\theta}=0 \tag{1a}$$

$$\frac{d\sigma_{\theta\theta}}{d\theta}+2\sigma_{r\theta}=0 \tag{1b}$$

2.2. Plane strain condition

If elastic response of the material is considered as incompressible then due to constancy of volume in plastic deformation the body is fully incompressible. Thus, plane strain condition is same for both elastic and plastic sectors and can be expressed as

$$\sigma_{33}=\frac{1}{2}(\sigma_{11}+\sigma_{22})=\frac{1}{2}(\sigma_{rr}+\sigma_{\theta\theta}) \tag{2}$$

2.3. Yield criterion

Von-Mises yield criterion for plane strain condition can be expressed as

$$\frac{1}{4}(\sigma_{rr}-\sigma_{\theta\theta})^2+\sigma^2{}_{r\theta}=k^2 \tag{3}$$

Here k is $\sigma_y/\sqrt{3}$ and σ_y is the yield strength in tension. The in-plane stress components in the plastic sector can be expressed in terms of a stress function $\psi(\theta)$ Zhu et al. [20]

$$\sigma_{rr}(\theta)=\sigma_m(\theta)-k\cos\psi(\theta) \tag{4a}$$

$$\sigma_{\theta\theta}(\theta)=\sigma_m(\theta)+k\cos\psi(\theta) \tag{4b}$$

$$\sigma_{r\theta}(\theta)=k\sin\psi(\theta) \tag{4c}$$