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# Nonlinear dynamic response of rotating circular cylindrical shells with precession of vibrating shape—Part I: Numerical solution

Y.Q. Wang, X.H. Guo*, H.H. Chang, H.Y. Li

*Institute of Applied Mechanics, Northeastern University, Shenyang 110004, China*

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ABSTRACT

The nonlinear dynamic response of a cantilever rotating circular cylindrical shell subjected to a harmonic excitation about one of the lowest natural frequency, corresponding to mode ($m$=1, $n$=6),where $m$ indicates the number of axial half-waves and $n$ indicates the number of circumferential waves, is investigated by using numerical method in this paper. The factor of precession of vibrating shape $\varsigma$ is obtained, with damping accounted for. The equation of motion is derived by using the Donnell's nonlinear shallow-shell theory, and is general in the sense that it includes damping, Coriolis force and large-amplitude shell motion effects. The problem is reduced to a system of ordinary differential equations by means of the Galerkin method. Three different mode expansions are studied for finding the proper one which is more contracted and accurate to investigate the principal mode (i.e., $m$=1, $n$=6) response. From the present investigation, it can be found that for principal mode resonant response, there are two traveling waves with different linear frequencies due to the effect of precession of vibrating shape of rotating circular cylindrical shells; the effects of additional modes $n$ and $k$ (multiples of frequency) on the principal mode resonant response are insignificant compared with an additional mode $m$, showing that it is better to adopt two neighboring axial modes to study the principal resonant response of the system.



## 1. Introduction

Rotating circular cylindrical shells are widely used in many industrial applications, such as gas turbine engines, electric motors, rotary kilns and rotor system. Hence, vibration characteristics of rotating cylindrical shells are of great importance. The precession of vibrating shape is a significant phenomenon for rotating cylindrical shells. When a circular cylindrical shell rotates about its centre axis, the vibrating shape shall not keep stationary, but move in a concentric circular path along the shell, which increases the difficulty to research the character of this structure. Some researchers have investigated the dynamics of rotating circular cylindrical shells; the first study is due to Bryan [1] who discovered the existence of precession of vibrating shape and obtained the expressions of the factor of precession for cirque and infinite length circular cylindrical shells. The effects of Coriolis and centrifugal forces on rotating shells have also been discussed by DiTaranto [2] and Huang [3]. Considering nonlinearity, Chen et al. studied finite length rotating cylindrical shells [4]. A method based on the use of the Love's first approximation theory was presented to study the free vibrations of a rotating truncated circular conical shell with simply supported boundary conditions by Lam and Li [5]. Ng et al. [6] first examined the parametric resonance phenomena in simply supported cylindrical shells. Using the generalized differential quadrature method, Li and Lam [7] carried out natural frequency analysis of thin rotating isotropic cylindrical shells. Critical speed of a rotating cylindrical shell with axial load has been studied by Ng and Lam [8]. Lee and Kim [9] studied the linear and nonlinear frequencies of a hybrid cylindrical shell by the Rize–Galerkin method. The vibration of rotating cross-ply laminated circular cylindrical shells with stringer and ring stiffeners was analyzed by Zhao et al. [10]. Liew et al. [11] proposed a meshfree method—the harmonic reproducing kernel particle (HRKP) method to study the effects of boundary conditions on the frequencies of rotating cylindrical shells, with the effects of the Coriolis and centrifugal force considered. The dynamic stability of composite laminated, functionally graded and rotating cylindrical shells under periodic axial forces were investigated by Liew et al., Ng et al. and Liew et al., respectively [12–14]. Using the wave propagation approach, the parametric analysis of frequency of rotating laminated composite cylindrical shells was studied by Zhang [15].

Noting the lack of published works on the dynamic response associated with the effect of precession of vibrating shape of rotating circular cylindrical shells, in the present study, the nonlinear response of a cantilever rotating circular cylindrical

* Corresponding author. Tel.: +86 024 83676166.
*E-mail address:* guoxinghui@mail.neu.edu.cn (X.H. Guo).

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**Nomenclature**

| | |
|---|---|
| $c$ | the coefficient of damping of the shell |
| $D$ | the flexural rigidity of the shell |
| $E$ | Young's modulus of the shell |
| $F$ | external excitation |
| $h$ | the wall thickness of the shell |
| Italic $\delta$ | Dirac delta function |
| $k$ | multiples of frequency |
| $L$ | the length of the shell |
| $m$ | the number of axial half-waves |
| $n$ | the number of circumferential waves |
| $R$ | the middle-surface radius of the shell |
| Roman $\mathrm{\delta}$ | variational symbol |
| $t$ | time |
| $\varsigma$ | the factor of precession of vibrating shape |
| $\rho$ | the mass density of the shell |
| $\mu$ | the Poisson ratio of the shell |
| $\varphi$ | the angle of precession of vibrating shape |
| $\varphi_1$ | rotary angle of the shell |
| $\Phi$ | Airy stress function |
| $\omega$ | radian frequency of external excitation |
| $\omega_{m,n}$ | the linear radian frequency corresponding to the mode $(m, n)$ |
| $\Omega$ | the angular velocity of the rotating shell |

shell, as shown in Fig. 1, with respect to the effect of precession of vibrating shape, is attempted to analyze. The factor of precession with respect to damping not considered by researchers before is obtained, which will be more accurate to predict the actual case of the vibrating shell. Based on the Donnell's nonlinear shallow-shell theory, nonlinear governing equation of the rotating circular cylindrical shell is derived, including the effects of the Coriolis force, damping and geometric large-amplitude. This paper is Part I of a series of papers which concern numerical solution of the governing equation. In particular, in order to reduce a drastic calculating effort, it is important to use only the most significant mode. The purpose of the present paper is to find the mode expansion which is more accurate and simpler describing the resonant response of rotating shells with precession of vibrating shape in the neighborhood of the principal mode ($m=1$, $n=6$).

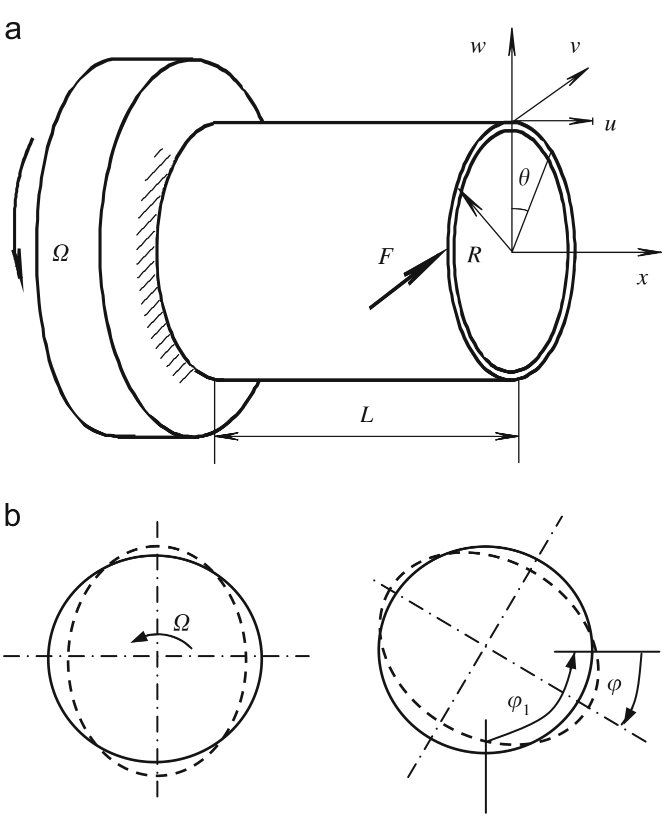


**Fig. 1.** (a) Coordinate system of a rotating cylindrical shell and (b) precession of vibrating shape of a rotating cylindrical shell.

## 2. Differential equation of motion

The circular cylindrical shell shown in Fig. 1(a) is considered to be thin, with length $L$, wall thickness $h$ and middle-surface radius $R$ and rotating with fixed edge at a constant angular velocity $\Omega$ about the $x$-axis. Its material properties are mass density $\rho$, the Poisson ratio $\mu$, the Young's modulus $E$ and the coefficient of damping $c$. A cylindrical coordinate system $(x, \theta, z)$ is chosen, with the origin placed at the centre of one end of the shell, where $x$ is the axial and $z$ is the radial coordinate. The displacements of points of the middle surface of the shell are denoted by $u$, $v$ and $w$, in the axial, circumferential and radial directions, respectively; $w$ is taken positive outwards. The harmonic excitation is assumed to be in the neighborhood of the mode $(m, n)$ of the shell having prevalent radial displacement. Fig. 1(b) shows the precession of vibrating shape phenomenon of a rotating shell, where the rotary angle $\phi_1$, the precession angle $\phi$ and the factor of precession of vibrating shape $\varsigma$ have the relationship $\phi=\varsigma\phi_1$.

Different from the Bryan's work [1], the effect of damping on the factor of precession $\varsigma$ is considered in this paper. The rotating of the shell about the $x$-axial denotes a movement, and can be regarded as a principle vibration, of which resonance frequency equal to zero. According to the summation of kinetic energy and potential energy keeping constant for principle vibration, and principle vibration not transferring energy to other principle vibrations, we get

$$\mathrm{\delta}\int_{t_0}^{t_1}\int_0^{2\pi}\int_0^L \frac{1}{2}\rho h\left[\left(\frac{\partial v}{\partial \varphi}\dot{\varphi}\right)^2+\left(\frac{\partial w}{\partial \varphi}\dot{\varphi}\right)^2\right]R\,\mathrm{d}\theta\,\mathrm{d}x\,\mathrm{d}t +2\Omega\rho h\int_{t_0}^{t_1}\int_0^{2\pi}\int_0^L\left(\frac{\partial v}{\partial \varphi}\dot{\varphi}\,\mathrm{\delta} w-\frac{\partial w}{\partial \varphi}\dot{\varphi}\,\mathrm{\delta} v\right)R\,\mathrm{d}\theta\,\mathrm{d}x\,\mathrm{d}t -c\int_{t_0}^{t_1}\int_0^{2\pi}\int_0^L\frac{\partial w}{\partial \varphi}\dot{\varphi}\,\mathrm{\delta} wR\,\mathrm{d}\theta\,\mathrm{d}x\,\mathrm{d}t=0. \tag{1}$$

where the symbol $\mathrm{\delta}$ is the variational symbol and is treated mathematically like a differential symbol, and here $\dot{\varphi}=\partial\varphi/\partial t$.

The following displacements $w$ and $v$, including the effect of the precession of vibrating shape, have been used

$$w(x,\theta,t)=W_m(x)[C_1(t)\cos n(\theta-\varphi)+C_2(t)\sin n(\theta-\varphi)], \tag{2}$$

$$v(x,\theta,t)=V_m(x)[C_1(t)\sin n(\theta-\varphi)-C_2(t)\cos n(\theta-\varphi)], \tag{3}$$

where $W_m(x)$ and $V_m(x)$ are the functions of axial vibrating shape of the shell, and $C_1(t)$ and $C_2(t)$ are unknown functions of time $t$.

For the circular cylindrical shell, the following relationships are introduced [16]:

$$\frac{\partial v}{\partial \theta}=-w, \tag{4a}$$

$$nV_m(x)+W_m(x)=0, \tag{4b}$$