International Journal of Mechanical Sciences 51 (2009) 642–652

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Vibration analysis of corrugated Reissner–Mindlin plates using a mesh-free Galerkin method

K.M. Liew [a,*], L.X. Peng [b], S. Kitipornchai [a]

[a] Department of Building and Construction, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China
[b] College of Civil Engineering and Architecture, Guangxi University, 530004 Nanning, China

ARTICLE INFO

Article history:
Received 23 July 2008
Received in revised form
18 March 2009
Accepted 13 June 2009
Available online 21 June 2009

Keywords:
Mesh-free galerkin method
Meshless
Corrugated plate
Free vibration
Equivalent properties

ABSTRACT

A mesh-free Galerkin method for the free vibration analysis of unstiffened and stiffened corrugated plates is introduced in this paper, in which the corrugated plates are simulated with an equivalent orthotropic plate model. To obtain the corresponding equivalent elastic properties for the model, a constant curvature state is applied to the corrugated sheet. The stiffened corrugated plates are treated as composite structures of equivalent orthotropic plates and beams, and the strain energies of the plates and beams are added up by the imposition of displacement compatible conditions between the plate and the beams. The stiffness matrix of the whole structure is then derived. The proposed method is superior to the finite element methods (FEMs) because no mesh is needed, and thus stiffeners (beams) do not need to be placed along the mesh lines and the necessity of remeshing when the positions of the stiffeners change is avoided. To demonstrate the accuracy and convergence of the proposed method, several numerical examples are analyzed both with the proposed method and the finite element commercial software ANSYS. Examples from other research are also employed. A good agreement between the results for the proposed method, the results of the ANSYS analysis, and the results from other research is observed. Both sinusoidally and trapezoidally corrugated plates are studied.

1. Introduction

Plates that are reinforced with corrugations (Fig. 1) can achieve a higher strength than flat plates, and can thus improve the strength/weight ratio of structures that are made of them. Because of these advantages, corrugated plates are popular materials for various engineering structures, such as decking, roofing, and sandwich plate cores.

The precise analysis of corrugated plates is quite complex and time consuming. The analysis of trapezoidally corrugated plates involves sheets lying in different planes and the transformation of every parameter that is expressed in the local coordinates of a sheet to a global coordinate. For sinusoidally corrugated plates, shell structures must be employed in the analysis, which requires a large amount of computation. A simple and valid way to study corrugated plates is to analyze them as orthotropic plates [1–12] of uniform thickness and equivalent rigidities (Fig. 2), and an approximated solution can thus be obtained. The approximation approach saves a great amount of effort with only a small loss of precision. Equivalent rigidities play a crucial role in the approximation approach, and their correct estimation is the key. The earliest estimation of equivalent rigidities is found in the work of Seydel [1], and for many years, researchers followed Seydel's formulas. Lau [6] improved on these formulas by deriving the theoretically correct form for the developed length l and moment inertia I. Briassoulis [7] studied the classical expressions [4,5], for equivalent rigidities, and obtained new and more precise expressions for the extensional rigidity and flexure rigidity of sinusoidally corrugated plates through the imposition of a constant strain state on the corrugated sheet. Shimansky and Lele [8] constructed an analytical model for the initial transverse stiffness of sinusoidally corrugated plates that incorporated the deformation that is caused by extension, shear, and bending. They obtained a simple approximate polynomial expression for the initial transverse stiffness of a thin plate, and found that transverse stiffness is not negligible for thick plates with a small degree of corrugation. Following the approach that was introduced by Briassoulis [7], Samanta and Mukhopadhyay [10] derived new expressions for the equivalent extensional rigidity of trapezoidally corrugated plates, and carried out a geometric nonlinear and free vibration analysis of such plates. To introduce shear deformation theory in the analysis of corrugated plates, Semenyuk and Neskhodovskaya [11] and Machimdamrong et al. [12] developed the equivalent expressions for the transverse shear modulus. Semenyuk and Neskhodovskaya [11] also discovered the instances when a

* Corresponding author.
E-mail address: kmliew@cityu.edu.hk (K.M. Liew).

0020-7403/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijmecsci.2009.06.005

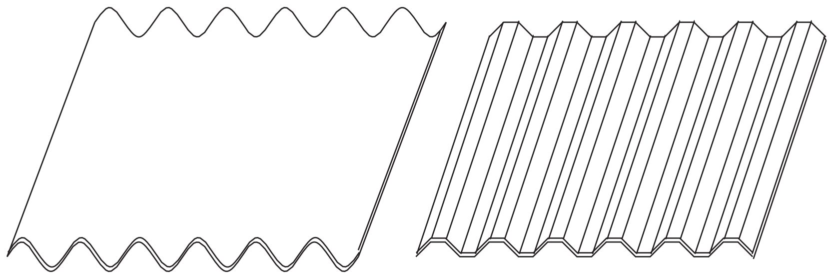

Fig. 1. Sinusoidally and trapezoidally corrugated plates.

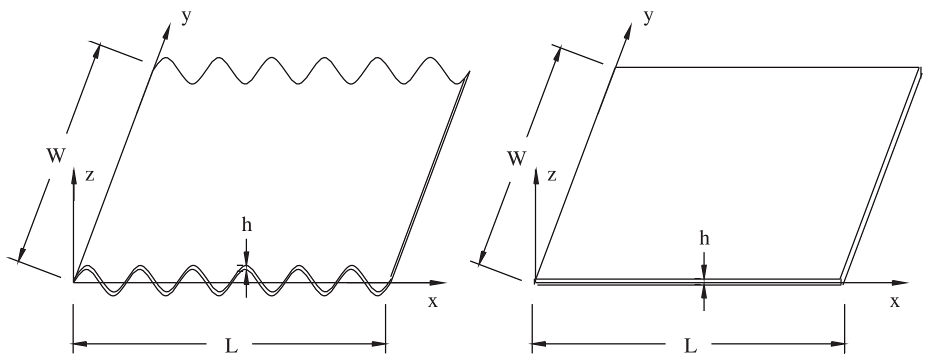

Fig. 2. Sinusoidally corrugated plate and its equivalent plate.

corrugated shell should not be treated as an equivalent of an orthotropic circular shell.

After the correct estimation of the equivalent rigidities, the remaining steps are much easier, and the only remaining task is the study of an equivalent orthotropic plate other than a fully corrugated plate, either by theoretical methods or numerical methods. Stiffened corrugated plates can also be considered as stiffened orthotropic plates. In the field of free vibration analysis of plates, a closed-form solution for initially stressed thick rectangular plates was presented by Xiang et al. [13]. The p-Ritz method was used to study the symmetrically laminated thick rectangular plates [14,15]. The differential quadrature method was employed to study circular [16] and rectangular plates [17,18]. All these works are contributed to the studies of dynamic behaviors of plates.

Due to their easy implementation with computers, numerical methods are very popular in engineering, and among those methods, the finite element methods (FEMs) are the most convenient, because they can be applied to large complex structures, and varied boundary and loading conditions can easily be applied. Nevertheless, the FEMs are not perfect, largely because they base their solutions on meshes. For dramatically large deformation and crack propagation problems, FEMs have difficulty in dealing with discontinuities that do not coincide with the original meshlines, and much effort is expended in remeshing at each step of the problem development. For stiffened plate problems, most FEMs require stiffeners to be placed along the meshlines, and any change in the position of the stiffeners means that the plate has to be remeshed. Because of these disadvantages, researchers have been searching for another powerful numerical tool as an alternative to the FEMs.

In recent years, some numerical methods that are known as meshless, or mesh-free, methods have gained more and more attention in the field [19–29]. Unlike the FEMs, the meshless methods construct their approximation solutions for problems entirely in terms of orderly or scattered points that are distributed on the domain of the problem structure that is being studied, and no other element or interrelationship is needed. Meshless methods are thus more applicable than FEMs to moving boundary problems, crack growth with arbitrary and complex paths, and phase transformation problems. Without the meshes, the aforementioned difficulties that are usually encountered by the FEMs disappear.

The objective of this paper is to introduce a mesh-free Galerkin method that is based on the first shear deformation theory (FSDT) for the free vibration analysis of unstiffened and stiffened corrugated plates. The corrugated plates are analyzed as equivalent orthotropic plates, and several numerical examples that are computed using the proposed method, the FEM software ANSYS, and the methods of other researchers are presented for comparison.

2. The mesh-free Galerkin method

By employing a moving least-square approximation, a function $v(\mathbf{x})$ in a domain Ω can be approximated by $v^h(\mathbf{x})$ in the subdomain $\Omega_{\mathbf{x}}$ and

$$v^h(\mathbf{x}) = \sum_{i=1}^{m} q_i(\mathbf{x}) b_i(\mathbf{x}) = \mathbf{q}^{\mathrm{T}}(\mathbf{x})\mathbf{b}(\mathbf{x}), \tag{1}$$

where $q_i(\mathbf{x})$ are the monomial basis functions, $b_i(\mathbf{x})$ are the corresponding coefficients, h is a factor that measures the domain of influence of the nodes, and m is the number of basis functions. A quadratic basis $\mathbf{q}^{\mathrm{T}} = [1, x, y, x^2, xy, y^2]$ $(m = 6)$ is used in this paper. The minimization of a weighted discrete L_2 norm

$$\Gamma = \sum_{I=1}^{n} \varpi(\mathbf{x} - \mathbf{x}_I)[\mathbf{q}(\mathbf{x}_I)^{\mathrm{T}}\mathbf{b}(\mathbf{x}) - v_I]^2 \tag{2}$$