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Instability and vibration of a rotating Timoshenko beam with precone

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ABSTRACT

A rotating blade with a precone angle is usually designed, but little literature has investigated the effect of the precone angle on vibration. This paper investigates divergence instability and vibration of a rotating Timoshenko beam with precone and pitch angles. It uses Hamilton's principle to derive the coupled governing differential equations and boundary conditions for a rotating Timoshenko beam. Analytical solution of an inextensional Timoshenko beam without taking into account the Coriolis force effect can be derived. Some simple relations among the parameters of rotating Timoshenko beams are revealed. Based on these relations, one can predict the natural frequencies and parameters of other systems from those of known systems. Moreover, the mechanism of divergence instability (tension buckling) is investigated. Finally, the effects of the parameters on natural frequencies, and the phenomenon of divergence instability are investigated.

1. Introduction

Rotating beams, which have importance in many practical applications such as turbine blades, helicopter rotor blades, airplane propellers, and robot manipulators, have been investigated for a long time. An interesting review of the subject can be found in the papers by Leissa [1], Ramamurti and Balasubramanian [2], Rosen [3], Lin [4,6], and Lin et al. [5,7]. Little attention has been focused on the investigation of the mechanism of instability because of its complexity. So far, no analytical solution for the vibration of a rotating beam with a precone angle has been presented.

Conventionally, Refs. [8–14] investigated the effects of tip mass, rotating speed, hub radius, pitch angle, taper ratio, shear deformation, rotary inertia, and elastic root restraints on the natural frequencies of transverse vibrations of a rotating beam. Regarding stability investigation, Lee and Lin [15] studied the vibration and the phenomenon of divergence instability of a rotating Timoshenko beam. Young and Lin [16] studied the stability of a cantilever tapered pretwisted beam with varying speed by using the Galerkin method. Kar and Neogy [17] used the Ritz method to study the stability of a rotating pretwisted cantilever beam. However, they did not investigate the effect of the precone angle on the stability and the mechanism of instability. Lin and Lee [18] studied the vibration and instability of a rotating frequency-dependent structurally and viscously damped beam with an elastically restrained root and root dampings. Lin [4] investigated the bending vibrations of a rotating Timoshenko beam. An analytical solution of this system was presented. Moreover, the influence of the parameters on natural frequencies, and the phenomenon of divergence instability were studied. But the effects of Coriolis force and the precone angle were not considered. Hodges and Ormiston [19] found that increasing the precone angle reduced the instability of a rotating beam. However, the complete relation between the instability and the precone angle was not discussed. Hosseini and Khadem [20] investigated the reliability of a rotating beam under random excitation. For preventing resonance, Maalawi and Negm [21] investigated the geometry design of a wind turbine blade with respect to the maximum frequency design criterion. The effect of the precone angle on the natural frequency was not clearly discussed. Lin et al. [5] investigated the vibration problem of a rotating Bernoulli–Euler beam with precone and pitch angles.

In this paper, the analytical methods given by Lin [4] and Lin et al. [5] will be used to solve the vibration problem of a rotating Timoshenko beam with precone and pitch angles. Moreover, the mechanism of divergence instability will be investigated.

2. Coupled governing equations and boundary conditions

Consider the flexural and axial motions of a rotating Timoshenko beam. The beam is elastically mounted with pitch angle θ and precone angle ϕ on a hub and rotates with constant

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Nomenclature

$A(x)$	cross-sectional area of the beam
$b(\xi)$	dimensionless bending rigidity $(=E(x)I(x)/E(0)I(0))$
$E(x)$	Young's modulus of beam material
$G(x)$	shear modulus of beam material
$g(\xi)$	dimensionless mass moment inertia $(=J(\xi)/J(0))$
$h(x)$	height of the beam
$I(x)$	area moment inertia of the beam
$J(x)$	mass moment of inertia of the beam per unit length
K_T	translational spring constants at the left end of the beam
K_θ	rotational spring constants at the left end of the beam
L	length of the beam
$m(\xi)$	dimensionless mass per unit length $(=\rho(\xi)A(\xi)/\rho(0)A(0))$
N	centrifugal force $(=\Omega^2\cos^2\phi\int_x^L \rho A(x+R)dx)$
$n(\xi)$	dimensionless centrifugal force $(=\alpha^2\cos^2\phi\int_\xi^1 m(\chi)(r+\chi)d\chi)$
$q(\xi)$	dimensionless shear rigidity $(=\kappa G(\xi)A(\xi)/\kappa G(0)A(0))$
T	kinetic energy
t	time variable
U	potential energy
u, v, w	displacements of the beam in the x, y, z-direction
u_0	axial neutral displacement due to axial force
$\vec{V}$	velocity
W	dimensionless displacement of the beam in the z-direction $(=\bar{w}/L)$
x	length variable of the beam
α	dimensionless rotating speed $(=\Omega L^2\sqrt{\rho(0)A(0)/[E(0)I(0)]})$
β_T	dimensionless translational spring constants $(=K_T L^3/[E(0)I(0)])$
β_θ	dimensionless rotational spring constants $(=K_\theta L/[E(0)I(0)])$
ε_{ij}	strain
ϕ	precone angle
γ_{1i}	dimensionless translational spring constant $(=\beta_i/(1+\beta_i),\ \ i=T,\theta)$
γ_{2i}	dimensionless rotational spring constant $(=1/(1+\beta_i),\ \ i=T,\theta)$
η	dimensionless ratio of mass moment inertia to mass at $x=0$ $(=J(0)/[\rho(0)A(0)L^2])$
κ	shear correction factor of the beam
Λ	dimensionless frequency $(=\omega L^2\sqrt{\rho(0)A(0)/[E(0)I(0)]})$
μ	dimensionless ratio of bending rigidity to shear rigidity at $x=0$ $(=E(0)I(0)/[\kappa G(0)A(0)L^2])$
θ	pitch angle
$\rho(x)$	mass density per unit volume
σ_{ij}	stress
Ω	rotating speed
ω	angular frequency of beam vibration
ξ	dimensionless distance to the root of beam $(=x/L)$
Ψ	angle of rotation due to bending

angular velocity Ω, as shown in Fig. 1. Describe the displacement vector of a point of the beam by $\vec{r}=(x+u)\vec{i}+(y+v)\vec{j}+(z+w)\vec{k}$ where $\{x,\ y,\ z\}$ are the coordinates of the point in the rotating beam frame and $\{u,\ v,\ w\}$ are its corresponding displacements. The displacement fields of the beam in the x-, y-, and z-directions are

$$u(x,z,t)=u_0(x,t)+z\Psi(x,t),\quad v=0,\quad w=w(x,t). \tag{1}$$

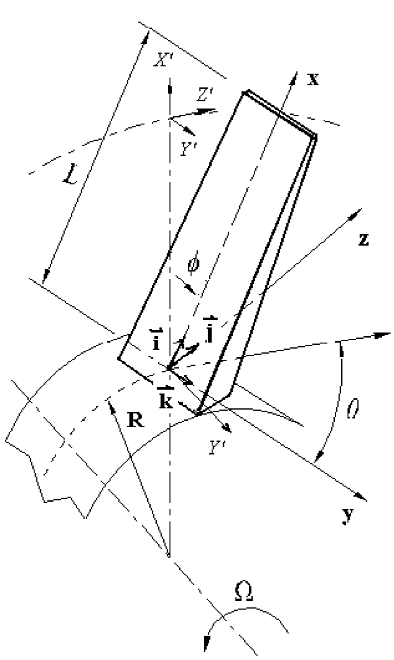

Fig. 1. Geometry and coordinate system of a rotating beam.

The velocity vector of the point $(x,\ y,\ z)$ in a beam is the derivative of the displacement vector and derived as

$$\begin{aligned}\vec{V}&=\frac{\partial u}{\partial t}\vec{i}+\frac{\partial w}{\partial t}\vec{k}+\vec{\Omega}\times\vec{r}\\&=\left[\frac{\partial u}{\partial t}+(z+w)\Omega\sin\theta\cos\phi+y\Omega\cos\theta\cos\phi\right]\vec{i}\\&\quad+[-(x+R+u)\Omega\cos\theta\cos\phi-(z+w)\Omega\sin\phi]\vec{j}\\&\quad+\left[\frac{\partial w}{\partial t}+y\Omega\sin\phi-(x+R+u)\Omega\sin\theta\cos\phi\right]\vec{k},\end{aligned} \tag{2}$$

where $\vec{\Omega}=\Omega(\sin\phi\,\vec{i}+\sin\theta\cos\phi\,\vec{j}+\cos\theta\cos\phi\,\vec{k})$.

The potential energy and the kinetic energy of a beam, respectively, are

$$U=\int_0^l\int_{-b/2}^{b/2}\int_{-h/2}^{h/2}\left(\frac{1}{2}\sigma_{xx}\varepsilon_{xx}+\sigma_{xz}\varepsilon_{xz}\right)dz\,dy\,dx+\frac{1}{2}k_\theta\Psi^2(0,t)+\frac{1}{2}k_T w^2(0,t) \tag{3}$$

and

$$T=\frac{1}{2}\int_0^L\int_{-b/2}^{b/2}\int_{-h/2}^{h/2}\vec{V}\cdot\vec{V}\rho\,dz\,dy\,dx. \tag{4}$$

Application of the Hamilton's principle yields the following governing differential equations:

$$\rho A\left[(x+R+u_0)\Omega^2\cos^2\phi+w\Omega^2\sin\beta\cos\theta\cos\phi-\frac{\partial^2 u_0}{\partial t^2}-2\frac{\partial w}{\partial t}\Omega\sin\theta\cos\phi\right]+\frac{\partial N}{\partial x}=0, \tag{5}$$

$$J\left(\Psi\Omega^2\cos^2\phi+\Omega^2\sin\phi\cos\theta\cos\phi-\frac{\partial^2\Psi}{\partial t^2}\right)+\frac{\partial}{\partial x}\left(EI\frac{\partial\Psi}{\partial x}\right)+\kappa GA\left(\frac{\partial w}{\partial x}-\Psi\right)=0, \tag{6}$$