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Vibration of skew plates by the MLS-Ritz method

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ABSTRACT

This paper presents a study on the vibration of skew plates by a numerical method, the moving least square Ritz (MLS-Ritz) method which was proposed by the authors in a previous study [Zhou L, Zheng WX. A novel numerical method for the vibration analysis of plates. Computational mechanics WCCM VI in conjunction with APCOM'04, Beijing, China, 5–10 September 2004; Zhou L, Zheng WX. MLS-Ritz method for vibration analysis of plates. Journal of Sound and Vibration 2006;290(3–5):968–90]. One of the most challenging numerical difficulties in analysing the vibration of a skew plate with a large skew angle is the slow convergence due to the stress singularities at the obtuse corners of the plate. The MLS-Ritz method is employed in this paper to address such problem. This method utilises the moving least square technique to establish the trial function for the transverse displacement of a skew plate and the Ritz method is applied to derive the governing eigenvalue equation for the skew plate. The boundary conditions of the plate are enforced through a point substitution technique that forces the MLS-Ritz trial function satisfying the essential boundary conditions along the plate edges. Due to the flexibility of the arrangement of the MLS-Ritz grid points, more grid points can be placed around the obtuse corners of a skew plate so as to address the stress singularity problem at the corners. A series of cases for rhombic plates of various edge support conditions are presented to demonstrate the efficiency and accuracy of the MLS-Ritz method.

1. Introduction

The study of buckling and vibration of skew plates dated back to the early 1950s when there was a need to investigate the mechanical properties of the then new swept-wing aircraft concept [3]. Skew plates are also widely employed in other practical structures, such as skew bridge decks, skew floor slabs, vehicle bodies and ship decks. Leissa [4] pointed out that no exact solutions exist for the vibration of skew plates and approximate numerical methods must be used to obtain solutions for such plates. A series of early investigations on the vibration behaviour of skew plates [5–9] were reviewed in Leissa's monograph [4]. Among these early studies, the Rayleigh-Ritz method was one of the most frequently used methods in analysing the vibration of skew plates [5–9]. Other methods used are Trefftz method [10], the perturbation method [11] and the point matching method [9].

The study on vibration of skew plates has attracted much attention since 1970s. Sathyamoorthy and Pandalai [12] employed the Berger approximation to study the relationship between the period and amplitude of skew plates based on an assumed mode shape. Mizusawa et al. [13] studied the free vibration of skew plates by the Rayleigh-Ritz method with B-spline functions as the Ritz trial functions to solve vibration of skew plates with arbitrary boundary conditions. It was found that, in general, the convergence of the vibration frequencies became less satisfactory with the increase in the skew angle of the plates. Mizusawa et al. [14] proposed a modified Rayleigh-Ritz method to analyse skew plates. Both geometric and natural boundary conditions were satisfied by using the Lagrange multiplier technique. Mizusawa and Kajita [15] also employed the spline strip method to investigate the vibration and buckling of skew plates with edges elastically restrained in rotation. A reduction method was proposed by Sakata [16] with a few approximation formulae for numerically estimating the natural frequency of simply supported isotropic and orthotropic skew plates. Gorman [17] studied the vibration of simply supported and clamped rhombic plates using the superposition method. Bardell [18] proposed a hierarchical finite element method to determine the natural frequencies and modes of flat, isotropic skew plates. The free edges and point supports were considered in his study.

Liew and Lam [19] employed the Rayleigh-Ritz method with 2D orthogonal plate functions as the Ritz trial function to study free vibration of skew plates. Rhombic plates with various combinations of edge support conditions were considered and good convergence and accuracy were demonstrated in their study. Liew and Wang [20] developed the pb-2 Ritz method to study the

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vibration of skew plates with different edge conditions, skew angles, aspect ratios and internal line supports. A comprehensive literature survey on the vibration of thin skew plates was presented in the paper. The pb-2 Ritz method was extended to study the buckling and vibration of thick skew plates based on the Mindlin shear deformable plate theory [21–23] and 3D elasticity [24,25]. Singh and Chakraverty [26] used the Rayleigh-Ritz method to determine the frequencies of skew plates with all possible combinations of boundary conditions and various skew angles. The boundary characteristic orthogonal polynomials were used to determine the transverse vibration of a rectangular or skew plate under different boundary conditions. Their results appeared not converged when the skew angle of the plates becomes large. Hadid and Bashir [27] employed the spline-integral method to calculate the natural frequencies of beams, rectangular and skew plates with different skew angles and simply supported edges. Han and Dickinson [28] studied the vibration of thin, symmetrically laminated skew plates by the Ritz method. Zitnan [29] studied the transverse vibration of rectangular and skew plates by the Rayleigh-Ritz method using B-spline trial functions. Recently, Woo et al. [30] carried out a study on the free vibration of skew Mindlin plates by employing the *p*-version of finite element method.

Although there are extensive studies on the vibration of skew plates in the open literature, the accuracy of the vibration solutions is not well addressed, especially for skew plates with large skew angles. It is due to the presence of strong stress singularity at the supported obtuse corners in the skew plates. The stress singularities of the skew plates lead to the difficulty of convergence when using numerical methods to determine accurate buckling, vibration and bending results for such plates [31]. Leissa and co-workers conducted a series of studies on free vibration of skew plates using the Ritz method in association with the corner stress singularity functions to address this problem [31–36]. Their studies showed that the inclusion of the stress singularity functions improves the convergence of vibration frequencies significantly and they were able to obtain accurate vibration frequencies for skew plates with large skew angles.

This paper employs the newly developed moving least square Ritz (MLS-Ritz) method [1,2] to analyse the vibration of rhombic plates. The purpose of the paper is to further verify and illustrate the MLS-Ritz method's validity and efficiency in dealing with rhombic plates of various combinations of edge support conditions and large skew angles. The influence of the MLS grind points on the convergence and accuracy of the method for analysing rhombic plates with large skew angles will be studied in details.

2. Mathematical modelling

Fig. 1 shows an isotropic, elastic skew plate of length a, width b, skew angle β and uniform thickness h in a Cartesian coordinate system. The plate is of the modulus of elasticity E, the Poisson ratio υ and the mass density ρ. The total potential energy functional of the plate based on the classical plate theory in harmonic vibration can be expressed as [4]:

$$F = \frac{D}{2}\int_A \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\upsilon)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} \mathrm{d}A - \frac{1}{2}\rho h \omega^2 \int_A w^2 \, \mathrm{d}A, \quad (1)$$

where $w(x,y)$ is the transverse displacement at the midsurface the plate, $D = Eh^3/[12(1-\upsilon^2)]$ is the flexural rigidity of the plate, A is the area of the plate and ω is the circular frequency of the vibration which needs to be determined.

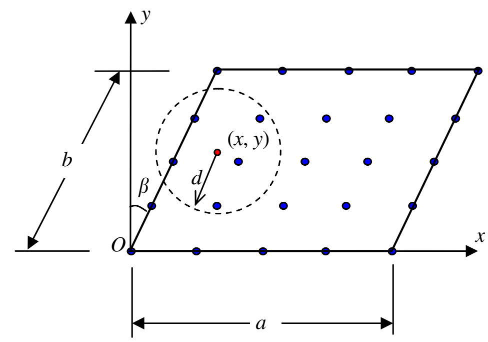

Fig. 1. Dimensions and coordinate system for a skew plate.

As the Ritz method is used to derive the governing eigenvalue equation for the plate system, only essential boundary conditions need to be satisfied:

- For a simply supported edge

$$w = 0 \quad (2)$$

- For a clamped edge

$$w = 0, \qquad \frac{\partial w}{\partial s} = 0, \quad (3)$$

where s is the normal direction to the edge. No constraint is needed for a free edge.

The MLS-Ritz method was developed by the authors and was applied to study the vibration of rectangular and triangular plates [2]. This method is briefly presented in this section for the consistence and easy reference. The Ritz trial function is first established through the MLS technique. A number of pre-determined points are selected on the calculation domain of the plate (see Fig. 1). The distribution of the points can be regular or irregular, depending on the requirement of the problem at hand.

Employing the MLS-Ritz method, the transverse displacement of a plate at an arbitrary point (x,y) can be approximated by the following expression [2]:

$$w(x,y) \approx \sum_{i=1}^{N} R_i(x,y) w_i = \mathbf{R}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}, \quad (4)$$

where N is the total number of grid points in the calculation domain, w_i in $\mathbf{w} = [\,w_1 \;\; w_2 \;\; \cdots \;\; w_n\,]^{\mathrm{T}}$ is the ith nominal value of displacement at (x_i, y_i), and the function $R_i(x,y)$ in $\mathbf{R} = [\,R_1(x,y) \;\; R_2(x,y) \;\; \cdots \;\; R_i(x,y) \;\; \cdots \;\; R_n(x,y)\,]$ can be determined as follows:

$$R_i(x,y) = \mathbf{p}^{\mathrm{T}}(x,y)\mathbf{A}^{-1} g_i(r)\mathbf{p}(x_i,y_i) \quad \text{if } r = \frac{\sqrt{(x-x_i)^2 + (y-y_i)^2}}{d} \leqslant 1, \quad (5)$$

$$R_i(x,y) = 0 \quad \text{if } r = \frac{\sqrt{(x-x_i)^2 + (y-y_i)^2}}{d} > 1, \quad (6)$$

in which d is the radius of domain of influence in the MLS interpolation (see Fig. 1), $\mathbf{p}(x,y) = [\,p_1(x,y) \;\; p_2(x,y) \;\; \cdots \;\; p_m(x,y)\,]^{\mathrm{T}}$ is a finite set of basis functions of a complete space, $g_i(r)$ is a weight function which takes the form as given below in this study:

$$g_i(r) = \begin{cases} (1-r^2)^k & \text{if } r \leqslant 1, \\ 0 & \text{if } r > 1, \end{cases} \quad (7)$$