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Thermoelastic lateral-torsional buckling of fixed slender beams under linear temperature gradient

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ABSTRACT

The thermal expansions and rotations that result from a linear in-plane temperature gradient field are fully restrained at the ends of a fixed beam. These restrained expansions and rotations will produce internal bending and compressive actions in the beam, and these actions increase with an increase of the temperature differential and average temperature of the linear temperature gradient field. When these actions reach critical values, the fixed beam may bifurcate from its primary equilibrium state to a buckled equilibrium configuration. This paper presents a systematic treatment of classical buckling analysis for thermoelastic lateral-torsional buckling and for in-plane thermoelastic flexural buckling of a fixed beam of doubly symmetric open thin-walled cross-section that is subjected to a linear temperature gradient field over its cross-section. It is shown that the effective centroid and shear centre, rather than the geometric centroid and shear centre, should be used in formulating the thermoelastic prebuckling and buckling analysis and that the effects of temperature on the buckling resistance need to be considered. The thermoelastic lateral-torsional buckling of a fixed beam under a linear temperature gradient field is more complicated than its mechanical counterpart for uniform bending or for uniform compression, and iterative methods are needed to obtain accurate solutions.

1. Introduction

When a beam is subjected to an in-plane temperature gradient field along the axis o^*y^* (Figs. 1a and b) that varies linearly from the bottom surface to the top surface of the cross-section, the heat tends to expand the fibres of the beam axially, i.e. in the direction of the axis o^*z^* and the possible expansions that result from this are also distributed linearly in-plane over the cross-section. This expansion gradient also tends to produce an in-plane curvature of the beam, and to cause the beam to deflect transversely in the plane of the temperature gradient [1–3]. For a fixed beam, its end thermal rotations are fully restrained; the restrained rotation will produce uniform bending in the fixed beam as shown by Gere and Timoshenko [1] and Gere [2]. For simplicity, the elastic modulus was treated in Refs. [1,2] as a constant over the entire beam, but it is well known that the elastic modulus of a material is a function of the temperature [5–7]. Because the temperature at a material point of the beam in a linear temperature gradient field is a function of the coordinates of the point, the elastic modulus at the point is also a function of its coordinates, and so the assumption of the elastic modulus of the fixed beam under a linear temperature gradient field as being a constant over the entire beam may lead to errors. In addition to uniform bending, a fixed beam is also subjected to an axial compressive action because the axial expansions of its ends are also fully prevented. Therefore, a fixed beam under a linear temperature gradient field is subjected to combined bending and compressive actions.

In the analysis of members under axial compression, the axial compressive force is usually assumed to act in the direction of the geometric centroidal axis if the beam is homogeneous. However, because the elastic modulus under a linear temperature gradient field is a function of the coordinates of the material point, the beam cannot be considered as homogeneous in resisting thermoelastic action and an axial force that acts at the geometric centroidal axis will produce additional bending moments in the beam. To avoid the complexity of these additional bending moments in the analysis, an effective centroidal axis needs to be determined such that when an axial compressive force acts in the direction of the effective centroidal axis, it produces pure axial compression [4]. In many ways this concept is identical to that of transformed areas in section composed of more than one material, but formulating this under thermal loading is complicated.

The thermal bending moment and axial compressive force in a fixed beam increase with an increase of the temperature differential and of the average temperature. When the combined bending and axial compressive actions reach a critical value, the fixed beam may suddenly deflect laterally and twist out of the plane of the temperature gradient and fail in a lateral-torsional

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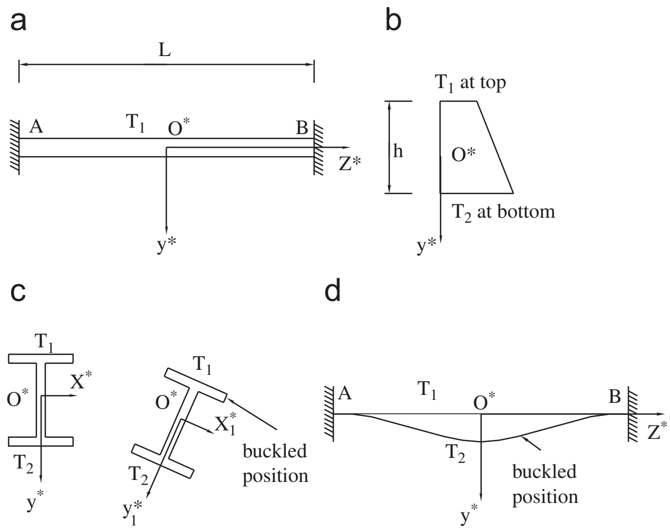

Fig. 1. Thermoelastic buckling of fixed beam: (a) fixed beam, (b) linear temperature gradient, (c) lateral-torsional buckling and (d) in-plane buckling.

bifurcation buckling mode or suddenly deflect in the plane of the temperature gradient and fail in a flexural buckling mode (Figs. 1c and d). In undertaking a lateral-torsional buckling analysis, an axis system with the shear centre (or centre of twist) as its origin is usually used [8], and in a mechanical elastic analysis, the shear centre of a doubly symmetric cross-section coincides with its geometric centroid [8]. However, in the thermoelastic buckling analysis of a fixed beam under a linear temperature gradient field, because the elastic modulus is a function of the temperature, it is not known whether the shear centre of the cross-section coincides with its geometric centroid or with its effective centroid. Because the elastic modulus is a function of the temperature distribution, the in-plane and out-plane bending, torsional, and warping stiffnesses are also functions of temperature distribution. In addition, a fixed beam under a linear temperature gradient field is subjected to combined bending and compressive actions. All of these factors make the thermoelastic buckling analysis of a fixed beam under a linear temperature gradient field much more complicated than that of a fixed beam under mechanical uniform bending or that of a fixed column under mechanical uniform compression.

The purpose of this paper is to present a systematic treatment of the classical buckling analysis for thermoelastic out-of-plane lateral-torsional and in-plane flexural buckling of a doubly symmetric open thin-walled section fixed beam that is under a linear in-plane temperature gradient field, and to derive the solution for the critical temperature gradient and critical average temperature for the thermoelastic buckling of the fixed beam based on this systematic analysis. In order to facilitate the investigation, formulas for determining the effective centroid, shear centre and the centre of twist of a cross-section under a linear temperature gradient field are also derived in this paper.

2. Effective centroid, shear centre, and centre of twist

2.1. Effective centroid

The following assumptions are used in this investigation for thermoelastic analysis:

1. Beams are assumed to be elastic and sufficiently slender, i.e. the ratio of their length to the dimensions of the cross-section is sufficiently large (for practical purposes over about 10 : 1) [5]. Deformations of these slender elastic beams can be assumed to satisfy the Euler–Bernoulli hypothesis, i.e. the cross-section remains plane and perpendicular to the beam axis during deformation.
2. The states of deformation and temperature are treated as time-independent, and so this separates the analysis of the temperature field from that of the displacement field and makes the problem uncoupled.
3. The temperature gradient is distributed linearly along the principal axis o^*y^* of the cross-section with temperatures T_1 and T_2 at the most top and bottom fibre of the cross-section (Fig. 2), but uniformly along the principal axis o^*x^* and the geometric centroidal axis o^*z^*, i.e. the temperature at an arbitrary point P is a linear function of its coordinate y^*, but not a function of its coordinates x^* and z^*. Hence, the temperature at an arbitrary point P can be expressed as

$$T(y^*) = T_{ave} + \frac{\Delta T y^*}{h}$$
$$\text{with } T_{ave} = \frac{T_1 + T_2}{2} \text{ and } \Delta T = T_2 - T_1, \tag{1}$$

where h is the overall height of the cross-section. Because the temperature gradient field is linear, the Euler–Bernoulli hypothesis holds during the thermal deformation.
4. The coefficient of thermal expansion α is independent of the temperature $T(y^*)$.
5. Because the thermoelastic analysis of slender beams is carried out with the same degree of rigour as that accepted in the theory of elasticity, expansions in the direction perpendicular to the beam axis are assumed to be so small that they can be disregarded in the analysis [9].

An arbitrary open thin-walled section shown in Fig. 2 is used in the derivation of the effective centroid, shear centre and centre of twist. When a beam is under a uniform temperature field, its modulus of elasticity is uniform over the entire beam. In this case, when an axial load is applied at the geometric centroid of the

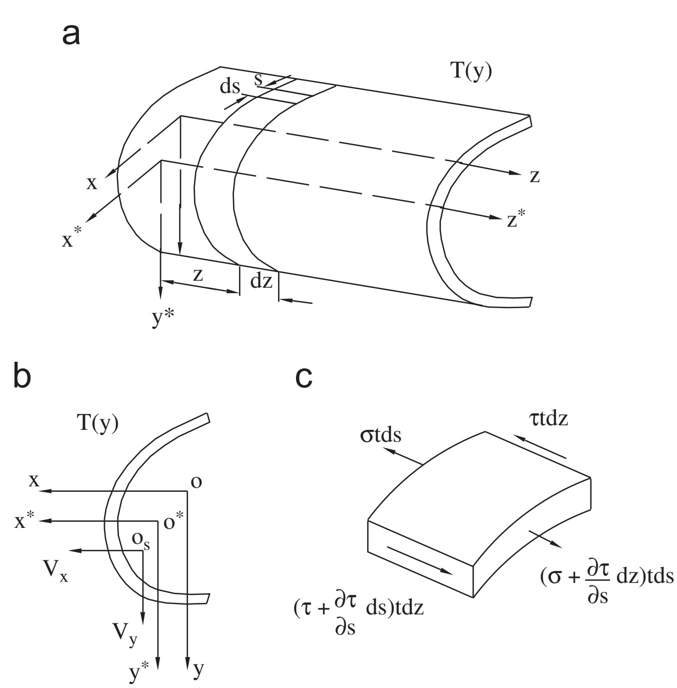

Fig. 2. Shear stresses due to pure bending: (a) thin-walled element, (b) vertical and horizontal shear forces and (c) stresses acting on infinitesimal element.