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Nonlinear free vibration of laminated composite plates on elastic foundation with random system properties

A. Lal [a,1], B.N. Singh [b,*], R. Kumar [c]

[a] Department of Applied Mechanics, MNNIT Allahabad, Allahabad 211004, India
[b] Department of Aerospace Engineering, Indian Institute of Technology, IIT Kharagpur, Kharagpur 721302, India
[c] Department of Applied Mechanics, MNNIT Allahabad, Allahabad 211004, India

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ABSTRACT

The present investigation is concerned with free vibration analysis of laminated composite plates resting on elastic foundation undergoing large amplitude oscillation with random system properties. The lamina material properties and foundation stiffness parameters are modeled as basic random variables for accurate prediction of the system behavior. The basic formulation of the problem is based on higher-order shear displacement theory including rotatory inertia effects and von Karman-type nonlinear strain displacement relations. A C^0 finite element is used for descretization of the laminate. A direct iterative method in conjunction with first-order Taylor series based perturbation technique procedure is developed to solve random nonlinear generalized eigenvalue problem. The developed probabilistic procedure is successfully used for the nonlinear free vibration problem with a reasonable accuracy. Typical numerical results (second-order statistics) are obtained for the composite plates resting on Winkler and Pasternak elastic foundations with different support conditions, side-to-thickness ratio, aspect ratio, oscillation amplitude ratio, stacking sequences and foundation parameters for symmetric and anti-symmetric cross-ply and angle-ply laminates. The results are validated with existing available results and independent Monte Carlo simulation.

1. Introduction

The study of large amplitude vibration of the composite plates which are widely used in engineering applications such as nuclear reactors, aircraft runways, building foundation slabs, indoor sport floors, etc., has attracted much interest from researchers because such vibration must be considered in designing resonance free composite structural components. Due to large number of parameters associated with the manufacturing and fabrication of composites and modeling of the foundation, the composite structures supported by elastic foundation display a considerable amount of uncertainties in their material and foundation properties. Therefore for accurate analysis of composite laminates supported on elastic foundation, it is necessary for engineers to estimate the probabilistic statistics of the eigensolutions of the structures for a reliable design.

Many publications have appeared in literature on the linear free vibration analysis of laminated composite plates [1–4] with deterministic system properties. Considerable literature is available on the nonlinear analysis of conventional structures and composite structures with and without elastic foundation with deterministic system properties [5–18].

The analysis of structures with deterministic characteristics to random excitation has been studied extensively in the literature [19]. However, the analysis of the structures with uncertain system properties is not developed to the same extent. Some literature is available on analysis of conventional structures and composite structures with random material properties. Salim et al. [20] have employed the first-order perturbation technique (FOPT) and Rayleigh–Ritz technique for analyzing composite plates in the framework of classical laminate theory with random material properties. They have modeled the material properties as random variables (RVs) and obtained the standard deviation (S.D.) of deflections of especially orthotropic composite laminates with deterministic loading having all edges simply supported. Singh et al. [21] have obtained the second-order statistics of the first five natural frequencies of cross-ply and angle-ply laminates using higher-order shear deformation theory (HSDT) with the FOPT. Raj et al. [22] have evaluated the stochastic static response of graphite–epoxy composite laminates using finite element method (FEM) with randomness in material properties subjected to deterministic loading. They have modeled the plate using HSDT and employed Monte Carlo simulation (MCS) to handle the randomness. They obtained the second-order statistics of the static response. Onkar and Yadav [23] have investigated nonlinear

* Corresponding author. Tel.: +91 3222 283026; fax: +91 3222 255303.
E-mail address: bnsingh@aero.iitkgp.ernet.in (B.N. Singh).
[1] Presently Lecturer, SVNIT Surat, India.

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free vibration response statistics of composite laminates with random material properties under random loading using a deterministic close form solution of the nonlinear problem in the framework of classical laminate theory in the von Karman sense. Zang and Chen [24] have presented a method to estimate the standard deviations of eigenvalue and eigenvector of random multiple degrees of freedom (MDOF) systems. The methods can be applied not only to the MDOF systems with distinct eigenvalues, but also to multi degree of freedom systems with repeated eigenvalues with uncertainties in mass and stiffness matrix elements. They have used both the sensitivity and the perturbation technique to develop the methodology. Zhangeen and Suhaun [25] have presented a method to estimate the standard deviation of eigenvalue and eigenvector of random MODF system. Zhang et al. [26] have applied the stochastic perturbation method to vector-valued and matrix-valued function for response and reliability of uncertain structures. Liu et al. [27] have studied the probabilistic finite element method for linear and nonlinear continua with inhomogeneous random fields. Zhang and Ellingwood [28] have evaluated the effect of random material field characteristics on the instability of a simply supported beam on elastic foundation and a frame using perturbation technique. Bruckner and Lin [29] have focused on generalization of the equivalent linearization method for nonlinear random vibration problems for certain statistical moments of the response. Benaroya and Rehak [30] examined the field of structural analysis where the FEM is used in a probabilistic setting.

It is evident from the literature that no work dealing with laminated composite plate resting on elastic foundation with random system properties is reported to the best of authors' knowledge. The contribution of this paper is the development of a HSDT based C^0 nonlinear PFEM combined with direct iterative method in conjunction with the FOPT for the nonlinear problem of laminated composite plates supported on elastic foundations with random system properties. The approach is valid for system properties with small random dispersions compared with the mean value. The condition is satisfied by most engineering materials as well as foundation and it hardly puts any limitation on the approach. And also the effect of the random changes in response on nonlinear stiffness parameters is neglected during iterative process with reasonable accuracy. The outlined approach has been used for evaluation of second-order statistics of the nonlinear free vibration of composite laminates supported on Winkler and Pasternak elastic foundations with known statistics of the basic random variables.

2. Formulations

The rectangular laminated composite plate of length a, width b, and thickness h, which consists of N plies, supported at four edges defined in (x, y, z) coordinate system with x- and y-axes located in the mid plane and its origin placed at the corner of the plate as shown in Fig. 1. The plate is assumed to attach to the foundation so that no separation takes place in the process of deformation. The load–displacement relation between the plate and the supporting foundation is as follows [15]:

$$P = K_1 w - K_2 \nabla^2 w \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{1}$$

where P, w, K_1 and K_2 are the foundation reaction per unit area, transverse displacement and normal (Winkler) and shear (Pasternak) foundation stiffnesses, respectively. This model is known as two-parameter Pasternak. When $K_2 = 0$, the model is reduced to the Winkler type.

2.1. Displacement field model

The displacement field in the framework of HSDT can be expressed as [1]

$$\begin{aligned} \bar{u}(x,y,z) &= u(x,y) + z\psi_x(x,y) + z^2\varsigma_x(x,y) + z^3\xi_x(x,y), \\ \bar{v}(x,y,z) &= v(x,y) + z\psi_y(x,y) + z^2\varsigma_y(x,y) + z^3\xi_y(x,y), \\ \bar{w} &= w, \end{aligned} \tag{2}$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ denote the displacements of a point along the (x, y, z) coordinates. u, v and w are corresponding displacements of a point on the mid plane. ψ_x and ψ_y are the rotations of normal to the mid plane about the y-axis and x-axis, respectively. The functions ς_x, ς_y, ξ_x and ξ_y are the higher-order terms in the Taylor series expansion defined in the mid plane of the plate. These functions are determined using the condition that the transverse shear stresses vanish on the top and bottom of the plate. Applying these conditions, the displacement field as given in Eq. (2) can be

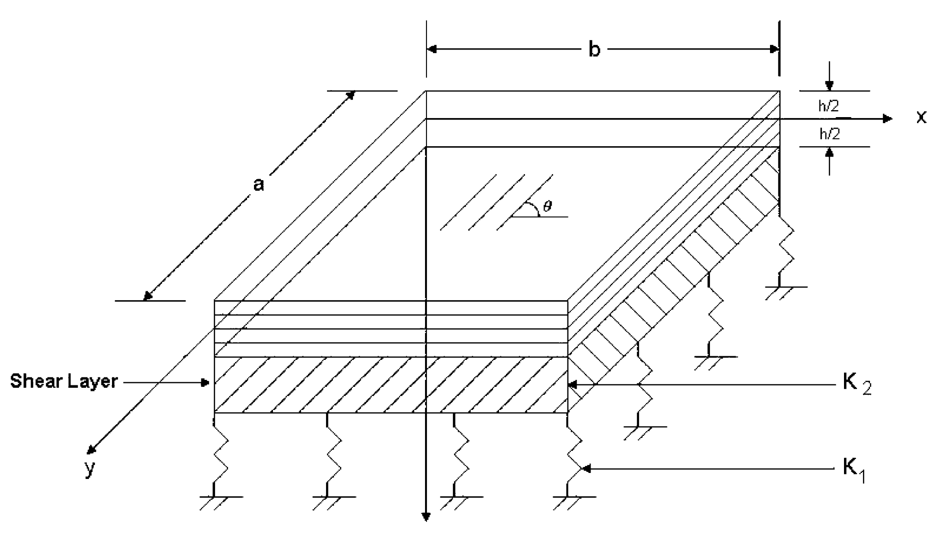

Fig. 1. Geometry of the laminated plate resting on elastic foundation.