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Dynamic stability of the viscoelastic rotating shaft subjected to random excitation

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Abstract

The dynamic stability problem of a viscoelastic Voigt–Kelvin rotating shaft subjected to action of axial forces at the ends is studied. The shaft is of circular cross-section, it rotates at a constant rate about its longitudinal axis of symmetry. The effect of rotatory inertia of the shaft cross-section is included in the present formulation. Each force consists of a constant part and a time-dependent stochastic function. Closed form analytical solutions are obtained for simply supported boundary conditions. By using the direct Liapunov method almost sure asymptotic stability conditions are obtained as the function of stochastic process variance, retardation time, angular velocity, and geometric and physical parameters of the shaft. Numerical calculations are performed for the Gaussian process with a zero mean and variance σ^2 as well as for harmonic process with amplitude H.

Keywords: Random loading; Liapunov functional; Almost sure stability; Rotatory inertia; Gaussian and harmonic process

1. Introduction

Rotating shafts, as elements of construction, often can take position to lose stability. The stability problem of rotating shafts arises when shafts are required to run smoothly at high speed. Destabilizing factors can be compressive force, the normal inertia force, as well as certain types of damping. So internal damping has this effect, while external damping generally has a stabilizing influence on the system.

The dynamic stability of rotating shafts, with omission of the compressive force, was first analyzed by Bishop [1] using a modal approach. The same problem using the direct Liapunov method was examined by Parks and Pritchard [2].

Shaw and Shaw [3] considered instabilities and bifurcations in non-linear rotating shaft made of viscoelastic Voigt–Kelvin material without compressive force.

Uniform stochastic stability of the rotating shafts, when the axial force is a wide-band Gaussian process with zero mean was studied by Tylikowski [4]. The rotating shaft subjected to axial forces with simultaneous internal damping (Voigt–Kelvin model) and external viscous damping was analyzed by the same author [5].

Tylikowski and Hetnarski [6] examined the influence of the activation through the change of the temperature on dynamic stability of the shape memory alloy hybrid rotating shaft.

Young and Gau [7,8] investigated dynamic stability of a pre twisted cantilever beam with constant and non-constant spin rates, subjected to axial random forces. By using stochastic averaging method, they determined mean-square stability condition in Ref. [7] and first and second moment stability conditions in Ref. [8].

In the present paper almost sure stability of the rotating viscoelastic Voigt–Kelvin shaft without accounting external damping is investigated. The axial force is stochastic process with known density function. Problem is solved by direct Liapunov method, and stability regions are given as function of geometric and physics parameters of the shaft.

2. Problem formulation

Let us consider a shaft rotating about its longitudinal axis with angular velocity $\bar{\Omega}$, shown in Fig. 1. In this Figure

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Notation

A	area of cross-section
I	axial moment of inertia
E	Young's modulus
f_{cr}	dimensionless Euler's critical force
f_o	dimensionless constant component of axial force
$f(t)$	dimensionless stochastic component of axial force
$\bar{F}$	axial force
l	length of the shaft
r	radius of gyration
p	probability density function
P	probability
t	dimensionless time
T	time
X, Y, Z	shaft coordinates
z	dimensionless axial shaft coordinate
u, v	flexural displacements in X and Y direction, respectively
V	Liapunov's functional
α_i	retardation time
ζ	dimensionless retardation time
$\bar{\Omega}$	angular velocity
Ω	dimensionless angular velocity
ρ	density
σ^2	variance of stochastic loading
$E\{\cdot\}$	mathematical expectation
$\|\|\cdot\|\|$	distance of solution from the trivial solution

(X, Y, Z) is rotating coordinate system where Z-axis coincides with longitudinal axis of the rotating shaft.

According to Young and Gau [7], governing differential equations can be written in the form

$$\rho A\left(\frac{\partial^2 u}{\partial T^2} - 2\bar{\Omega}\frac{\partial v}{\partial T} - \bar{\Omega}^2 u\right) - \rho I\,\frac{\partial^4 u}{\partial T^2 \partial Z^2} + EI\alpha_i \frac{\partial^5 u}{\partial T \partial Z^4} + EI\frac{\partial^4 u}{\partial Z^4} + \bar{F}(T)\frac{\partial^2 u}{\partial Z^2} = 0, \tag{1}$$

$$\rho A\left(\frac{\partial^2 v}{\partial T^2} + 2\bar{\Omega}\frac{\partial u}{\partial T} - \bar{\Omega}^2 v\right) - \rho I\,\frac{\partial^4 v}{\partial T^2 \partial Z^2} + EI\alpha_i \frac{\partial^5 v}{\partial T \partial Z^4} + EI\frac{\partial^4 v}{\partial Z^4} + \bar{F}(T)\frac{\partial^2 v}{\partial Z^2} = 0, \tag{2}$$

where u, v are flexural displacements in the X and Y direction, ρ is mass density, A is area of the cross-section of shaft, I is axial moment of inertia, E is Young modulus of elasticity, α_i is retardation time, T is time and Z is the axial coordinate.

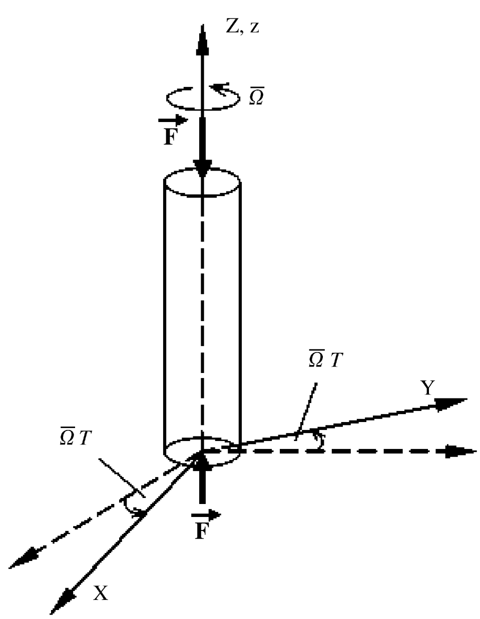

Fig. 1. The rotating shaft and co-ordinate systems.

Using the following transformations:

$$Z = z\ell, \quad e^2 = \frac{I}{A\ell^2}, \quad k_t = \sqrt{\frac{\rho A \ell^4}{EI}}\,T = k_t t,$$

$$f_0 + f(t) = \frac{\bar{F}(t)\ell^2}{EI}, \quad 2\varsigma = \frac{\alpha_i}{k_t}, \quad \Omega = \bar{\Omega}k_t, \tag{3}$$

where ℓ is the length of the shaft and ζ is reduced retardation time, we get governing equations as

$$\frac{\partial^2 u}{\partial t^2} - 2\Omega\,\frac{\partial v}{\partial t} - \Omega^2 u - e^2\,\frac{\partial^4 u}{\partial t^2 \partial z^2} + 2\varsigma\,\frac{\partial^5 u}{\partial t\,\partial z^4} + \frac{\partial^4 u}{\partial z^4} + (f_0 + f(t))\frac{\partial^2 u}{\partial z^2} = 0, \tag{4}$$

$$\frac{\partial^2 v}{\partial t^2} + 2\Omega\,\frac{\partial u}{\partial t} - \Omega^2 v - e^2\,\frac{\partial^4 v}{\partial t^2 \partial z^2} + 2\varsigma\,\frac{\partial^5 v}{\partial t \partial z^4} + \frac{\partial^4 v}{\partial z^4} + (f_0 + f(t))\frac{\partial^2 v}{\partial z^2} = 0, \tag{5}$$

$z \in (0,\ 1)$.

Boundary conditions for the simply supported shaft are

$$u(t,0) = u(t,1) = \frac{\partial^2 u}{\partial z^2}(t,0) = \frac{\partial^2 u}{\partial z^2}(t,1) = 0,$$
$$v(t,0) = v(t,1) = \frac{\partial^2 v}{\partial z^2}(t,0) = \frac{\partial^2 v}{\partial z^2}(t,1) = 0. \tag{6}$$

The purpose of the present paper is the investigation of almost sure asymptotic stability of the rotating shaft subjected to stochastic time-dependent axial loads. To estimate perturbated solutions it is necessary to introduce a measure of distance $\|\cdot\|$ of solutions of Eqs. (4) and (5) with nontrivial initial conditions and the trivial one. Following Kozin [9], the equilibrium state of Eqs. (4) and