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Computational implementation of a non-linear kinematic hardening formulation for tension–torsion multiaxial fatigue calculations

Hao Wu [a], Marco Antonio Meggiolaro [b,*], Jaime Tupiassú Pinho de Castro [b]

[a] School of Aerospace Engineering and Applied Mechanics, Tongji University, 1239 Siping Road, Shanghai, PR China
[b] Department of Mechanical Engineering, Pontifical Catholic University of Rio de Janeiro, Rua Marquês de São Vicente 225 – Gávea, Rio de Janeiro, RJ 22451-900, Brazil

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ABSTRACT

The calculation of elastoplastic strains from stress histories, or vice-versa, is an important computational step in low-cycle fatigue analyses. This step is a challenging task for general multiaxial non-proportional (NP) loading histories, where the principal stress directions are not constant, requiring 6D incremental plasticity calculations to correlate the six stress with the six strain components considering plasticity effects. However, a large number of multiaxial fatigue problems only involve combined tension and/or bending and torsion loads, which are associated with only one normal and one shear stress component. The use of a special 2D formulation, instead of 6D, can greatly simplify the necessary incremental plasticity calculations for these practical problems. In this work, a new 2D tension–torsion incremental plasticity formulation is introduced, integrating non-linear kinematic (NLK) hardening models and NP hardening effects in a very efficient way, exactly reproducing tension–torsion calculations from more general 6D models, but with less than one fifth of the computational cost. The proposed 2D approach is validated by comparing NP strain-controlled tension–torsion experiments in 316L steel tubular specimens, a material that presents significant NP hardening effects, with experimental and predicted stress paths, calculated either with 6D or the proposed 2D formulation.

1. Introduction

Most engineering applications involve either known stress or strain histories, but not both. New components are normally designed based on stress histories calculated or estimated from measured or specified design loads, whereas advanced structural integrity evaluations use strain histories properly measured in the field under actual service conditions; since stresses cannot be measured, they can only be calculated. However, most multiaxial fatigue models require both the stress and the strain histories to quantify the damage induced by the loading history.

Calculation of multiaxial stresses from given strains or vice-versa is a trivial task if the load at the critical point is linear elastic (LE), only requiring the application of Hooke's law. But for low-cycle fatigue calculations, where cyclic plasticity effects can be very significant, incremental plasticity models are usually needed to correlate multiaxial stresses and strains, especially under variable amplitude (VA) non-proportional (NP) loadings. Two approaches can be followed in these cases to estimate crack initiation lives: performing global elastoplastic (EP) incremental finite element (FE) calculations for the entire component along the loading history, a computationally prohibitive task for VA load histories with many cycles or events; or instead use a much simpler global–local approach [1–5], where a single LE FE calculation on the entire piece is performed for a static unit value of each applied loading, followed by local incremental plasticity calculations at every load step only at the critical point(s), to correct for plasticity effects.

The former approach requires global Finite Element (FE) calculations to evaluate the interaction among EP stresses and strains, considering as well stress gradient effects near the critical point. This global EP FE approach thus needs to adopt an incremental plasticity formulation in every element of the mesh that represents the studied structural component that suffers plastic strains. This requirement is computationally very intensive, especially when dealing with long loading histories, since it implies in having to solve the EP FE problem for the entire piece for every load increment of every load cycle (or of every load event in complex VA cases, where cycles cannot be identified).

The global–local approach, on the other hand, can be very accurate and computationally much more efficient if carefully performed, as described next. Consider a general case of N applied

* Corresponding author. Tel.: +55 21 3527 1424; fax: +55 21 3527 1165.
E-mail addresses: wuhao@tongji.edu.cn (H. Wu), meggi@puc-rio.br (M.A. Meggiolaro), jtcastro@puc-rio.br (J.T.P. de Castro).

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loads (which could be e.g. forces, moments, or displacements), with time histories given by $F_1(t), F_2(t), \ldots$, see Fig. 1. A constant unit load $F_1(t) = 1$ is imposed to the piece in a LE FE calculation, to obtain the resulting six stress and strain components at the critical point, which would constitute the LE stress and strain influence factors $K_{\sigma x1}, K_{\sigma y1}, K_{\sigma z1}, K_{\sigma xy1}, K_{\sigma xz1}, K_{\sigma yz1}, K_{\varepsilon x1}, K_{\varepsilon y1}, K_{\varepsilon z1}, K_{\varepsilon xy1}, K_{\varepsilon xz1}$, and $K_{\varepsilon yz1}$, see Fig. 1. Another LE FE calculation with only $F_2(t) = 1$ would then obtain the associated stress and strain influence factors $K_{\sigma x2}, K_{\sigma y2}, \ldots, K_{\varepsilon yz2}$, and so on. These factors can then be organized into LE stress and strain influence matrices, as shown in Fig. 1, which calculate the so-called pseudo-stresses and pseudo-strains (represented with a tilde "∼" mark, see Fig. 1), i.e. stresses and strains assumed to be LE, even though they in general might not be elastic and thus later require elastoplastic corrections.

These matrices are then used to compute the so-called pseudo-stresses and pseudo-strains which, as mentioned above, are fictitious quantities calculated assuming the material follows Hooke's law at the critical point of the piece. The pseudo-stresses $\vec{\tilde{\sigma}}$ and strains $\vec{\tilde{\varepsilon}}$ are obtained after multiplying the actual value of the loadings $F_1(t), F_2(t), \ldots$, at each instant t by their associated LE influence factors, and adding them at the critical point using the superposition principle.

If the critical point is not evident, then these matrices need to be calibrated for each potential critical location. The potential location that results in the highest accumulated multiaxial fatigue damage is then the critical point where the crack is expected to initiate. The direction of such a crack could also be calculated using the critical-plane approach for all candidate planes at this critical point [6].

The resulting pseudo-histories $\vec{\tilde{\sigma}}(t)$ and $\vec{\tilde{\varepsilon}}(t)$ can deal with multiple in- or out-of-phase loading sources applied to the structural component, but they are LE values that in general still require EP corrections to reproduce the true stresses and strains at its critical point. Indeed, in their LE form they can only be used in the absence of significant macroscopic plasticity at the critical point, i.e. they are only useful for high-cycle fatigue calculations that do not involve residual stresses induced by eventual overloads. Otherwise, a proper multiaxial incremental plasticity formulation must be used to account for cyclic kinematic, isotropic, and NP hardening effects in the EP stress/strain behavior, in general considering notch stress and strain concentration effects [1–5]. However, the involved calculations require the solution of a set of dozens of stiff differential equations, a challenging task that prevents its widespread use in engineering problems without the aid of advanced and dedicated commercial fatigue software, thus involving costs that are usually prohibitive for small companies.

In the following sections, a new simplified yet accurate incremental plasticity formulation is fully developed for combined tension–torsion problems, a very important practical case induced by normal and/or bending and torsional loads in common components such as shafts and beams. It is shown that the full 6D stress–strain problem does not need to be solved in such cases, which can be managed using a 2D reduced-order formulation that much simplifies its computer implementation and also reduces the calculation time in more than 80%.

2. Tension–torsion hardening formulation

Tension–torsion incremental plasticity calculations are most efficiently performed in a 2D stress space $\sigma_x \times \tau_{xy}\sqrt{3}$, where σ_x is the normal and $\tau_{xy}\sqrt{3}$ is the effective shear stress. The yield surface is defined as the root locus of the stress states $\vec{s} = [\sigma_x \quad \tau_{xy}\sqrt{3}]^T$ (or stress points, where T stands for transpose of a vector or matrix) where the material starts to yield. Under tension–torsion, the yield surface can be described as a circle in this diagram if the material follows the von Mises criterion, since

$$\vec{s} = [\sigma_x \quad \tau_{xy}\sqrt{3}]^T \Rightarrow Y = |\vec{s}|^2 - S^2 = \left(\sigma_x^2 + 3\tau_{xy}^2\right) - S^2 = 0 \qquad (1)$$

where S is the current radius of the yield surface, e.g. the monotonic yield strength S_Y for a tensile test or the cyclic yield strength S_{Yc} for a cyclically-stabilized loading, and $Y = 0$ is the yield function.

2.1. Kinematic hardening formulation

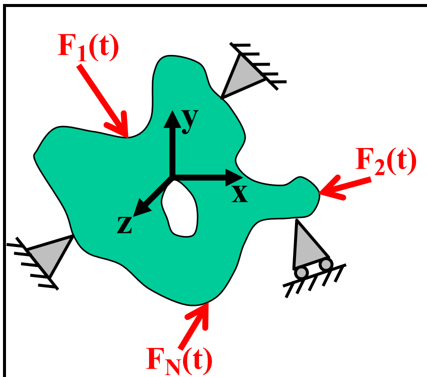
Loading a piece above its yield limit in one direction reduces (in absolute value) its yield strength in the opposite direction, a phenomenon known as the Bauschinger effect. For a tension–torsion history, this effect can be represented as a translation of the yield surface, whose center translates to a so-called backstress position $\vec{\beta} = [\beta_x \quad \beta_{xy}\sqrt{3}]^T$, see Fig. 2 [7]. This backstress vector is the quantity that stores the plastic memory effects required for kinematic hardening calculations.

Assuming the normal stresses in the x direction, if the material is isotropic and thus has symmetry in the y and z directions, then its elastic (el) and plastic (pl) strain components can also be represented in 2D, by

$$\vec{e}_{el} \equiv (1+\nu) \cdot \left[\varepsilon_{x_{el}} \quad \frac{\gamma_{xy_{el}}}{2\cdot(1+\nu)}\sqrt{3}\right]^T \text{ and } \vec{e}_{pl} \equiv \frac{3}{2} \cdot \left[\varepsilon_{x_{pl}} \quad \frac{\gamma_{xy_{pl}}}{\sqrt{3}}\right]^T \qquad (2)$$

where ν is Poisson's ratio, ε stands for normal and γ for shear strains.

This efficient representation is a 2D strain sub-space from the 5D deviatoric space introduced in [8] and further detailed in [9].



$$\begin{bmatrix}\tilde{\sigma}_x(t)\\ \tilde{\sigma}_y(t)\\ \tilde{\sigma}_z(t)\\ \tilde{\tau}_{xy}(t)\\ \tilde{\tau}_{xz}(t)\\ \tilde{\tau}_{yz}(t)\end{bmatrix} = \begin{bmatrix} K_{\sigma x1} & K_{\sigma x2} & \cdots & K_{\sigma xN}\\ K_{\sigma y1} & K_{\sigma y2} & \cdots & K_{\sigma yN}\\ K_{\sigma z1} & K_{\sigma z2} & \cdots & K_{\sigma zN}\\ K_{\sigma xy1} & K_{\sigma xy2} & \cdots & K_{\sigma xyN}\\ K_{\sigma xz1} & K_{\sigma xz2} & \cdots & K_{\sigma xzN}\\ K_{\sigma yz1} & K_{\sigma yz2} & \cdots & K_{\sigma yzN}\end{bmatrix} \cdot \begin{bmatrix}F_1(t)\\ F_2(t)\\ \vdots\\ F_N(t)\end{bmatrix}$$

$$\begin{bmatrix}\tilde{\varepsilon}_x(t)\\ \tilde{\varepsilon}_y(t)\\ \tilde{\varepsilon}_z(t)\\ \tilde{\gamma}_{xy}(t)\\ \tilde{\gamma}_{xz}(t)\\ \tilde{\gamma}_{yz}(t)\end{bmatrix} = \begin{bmatrix} K_{\varepsilon x1} & K_{\varepsilon x2} & \cdots & K_{\varepsilon xN}\\ K_{\varepsilon y1} & K_{\varepsilon y2} & \cdots & K_{\varepsilon yN}\\ K_{\varepsilon z1} & K_{\varepsilon z2} & \cdots & K_{\varepsilon zN}\\ K_{\varepsilon xy1} & K_{\varepsilon xy2} & \cdots & K_{\varepsilon xyN}\\ K_{\varepsilon xz1} & K_{\varepsilon xz2} & \cdots & K_{\varepsilon xzN}\\ K_{\varepsilon yz1} & K_{\varepsilon yz2} & \cdots & K_{\varepsilon yzN}\end{bmatrix} \cdot \begin{bmatrix}F_1(t)\\ F_2(t)\\ \vdots\\ F_N(t)\end{bmatrix}$$

Fig. 1. FE-calibrated linear elastic matrices correlating several applied scalar load histories with the twelve resulting pseudo-stress and pseudo-strain histories at the critical point.