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Multiaxial variable amplitude fatigue life analysis including notch effects

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ABSTRACT

This paper evaluates the methodology for, and effectiveness of, common equivalent stress- and strain-based fatigue life analysis approaches when applied to variable amplitude multiaxial service loading conditions. Experimental results for both un-notched and notched specimens of 2024-T3 aluminum alloy, tested under axial, torsion, and combined axial–torsion loadings, are compared to predictions based on von Mises equivalent stress and strain. Mean stresses were considered using Smith–Watson–Topper, modified Goodman, and modified Morrow models. Variable amplitude life predictions are compared to constant amplitude predictions to highlight similarities and differences in life prediction trends. Overall, mixed results were obtained. Both constant and variable amplitude fatigue lives are predicted well for notched specimens, where a local uniaxial stress state always exists for the geometry considered. However, for un-notched specimens, where multiaxial loading effects come into play, no approach considered is able to predict more than 60% of fatigue data within a factor of ±3 of experimental life. A consistent trend of non-conservative life predictions for variable amplitude loading conditions is observed. The importance of considering stress gradient effects in notched specimen life prediction is also demonstrated. An extensive discussion on areas where improvements can be made with respect to fatigue damage quantification completes the paper.

1. Introduction

Most engineering components and structures are subjected to variable amplitude cyclic loadings throughout their service lives. Due to the nature of these loadings, they typically result in multiaxial stress states, and individual stress components can vary in a non-proportional manner. Additionally, geometrical notches, which are often unavoidable in practice, result in localized regions of elevated stress and can alter the local stress state within these components. However, despite the significance of such conditions, the synergistic complexity of fatigue crack initiation with multiaxial stresses and stress concentrations under variable amplitude loadings has only been evaluated in a very small number of studies (e.g. [1–3]). Available experimental evidence suggests that some commonly used fatigue damage analysis techniques may not be capable of producing accurate life predictions for such complex and yet highly practical conditions.

Even in the absence of a notch, fatigue life analyses under multiaxial variable amplitude loadings can be quite complex. Due to relatively limited amounts of research and experimental data for such conditions, no generally accepted procedures exist for performing such an analysis. Of the methods that are available, they are often complex and time consuming to implement. Therefore, to simplify the analysis, classical equivalent stress or strain approaches, such as those based on von Mises, maximum shear, and maximum principal stress yield criteria, are commonly extended to situations involving multiaxial loading histories. Stress-based approaches are typically used in situations where the material behavior is primarily elastic, while strain-based approaches can better account for the presence of plastic deformation [4]. If mean stresses are present, an equivalent mean stress, to be used in a uniaxial mean stress correction model, may be computed using criteria such as von Mises effective mean stress or hydrostatic stress. For non-proportional loadings, however, these equivalent stress approaches fail to account for the increase in damage and/or non-proportional hardening resulting from the rotation of principal stress directions. As a result, alternative equivalent stress-based approaches have been proposed by Sonsino [5] and Lee et al. [6].

Because loading components remain in a constant ratio to one other under linear elastic proportional loading, cycle counting may be performed on any stress or strain component, or an equivalent stress/strain history, and the range and mean of each stress/strain component can subsequently be determined through

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Nomenclature

b	axial fatigue strength exponent	$\Delta\varepsilon_q$	equivalent strain range
c	axial fatigue ductility exponent	$\Delta\sigma_q$	equivalent stress range
E	modulus of elasticity	$\Delta\sigma_q^e$	equivalent elastic (pseudo) stress range
K'	cyclic axial strength coefficient	ρ	material characteristic length
K'_o	cyclic shear strength coefficient	σ'_f	axial fatigue strength coefficient
K_f	fatigue notch factor	σ_{qa}	equivalent stress amplitude
K_t	elastic stress concentration factor	σ_{qm}	mean equivalent stress
n'	cyclic axial strain hardening exponent	$\sigma_{q,max}$	maximum equivalent stress
n'_o	cyclic shear strain hardening exponent	σ_u	ultimate strength
N_f	cycles to failure	σ_y	tensile yield strength
r	notch root radius	σ'_y	cyclic yield strength
S_{Nf}	effective fully-reversed stress amplitude		
ε'_f	axial fatigue ductility coefficient		
ε_{qa}	equivalent strain amplitude		

appropriate scaling of the counting variable. For non-proportional loading, however, loading components may be applied out-of-phase, at different frequencies (asynchronous), or randomly with respect to one another. In the case of asynchronous and/or random loadings, which are often encountered in service loading histories, there may be no clear definition of a cycle. Therefore, the amplitudes and mean stresses of the different loading components, required as input for the equivalent stress/strain approaches, can change depending on the component used for cycle counting. This is true even for the equivalent non-proportional stress criteria of Sonsino and Lee et al., thus making the application of these approaches vague under such conditions.

One way to deal with this issue is to compute equivalent stress/strain values first, and then cycle count the single equivalent stress/strain history. However, there are a number of drawbacks to this approach. The most obvious is the need to assign a positive or negative value to the equivalent stress quantity, so that stress ranges and reversals can be properly identified. Although there are multiple methods of doing this, the most common is to use the sign corresponding to the principal stress with the highest absolute value [7]. However, in the case of non-proportionally varying stress components, this has the potential to produce stress histories which are not representative of the actual fatigue damage experienced by a component [8]. An easy way to illustrate this point is to consider the case of axial–torsion loading. A look at Mohr's circle reveals that, in this situation, the sign of the equivalent stress quantity is always equal to that of the axial stress component. Based on this fact, Fig. 1 shows a short loading history in terms of applied axial stress, shear stress, and the resulting signed von Mises stress. When looking at this figure, if the small axial stress cycles are considered insignificant compared to the much larger shear cycle, one would expect the predicted fatigue damage (from von Mises equivalent stress) to be representative of that due to the single shear cycle. However, due to the presence of the axial cycles, the computed equivalent stress history predicts several large magnitude cycles which result in significant fatigue damage. The same would also be true for the common case of combined bending–torsion loading. Therefore, the application of the signed equivalent stress approach, in this case, would lead to overly conservative fatigue life predictions.

Additionally, the consideration of mean stress effects for a signed equivalent stress history loses some physical relevance. In traditional equivalent stress-based approaches, an equivalent mean stress is computed from the mean values of the normal, or principal, stress components over a given cycle. This mean stress value is then used, along with the equivalent stress amplitude, in traditional uniaxial mean stress correction models to compute a new equivalent fully-reversed stress amplitude from which fatigue damage is computed. However, for a signed equivalent stress history, the mean value of the equivalent stress quantity is used directly in the mean stress corrections. In the case of von Mises stress, this can result in the prediction of increased fatigue damage due to the presence of mean shear stresses. However, mean shear stresses have been shown to have little effect on fatigue damage so long as they remain below the material yield strength [9].

An alternative approach to performing variable amplitude equivalent stress/strain fatigue analysis is to use the multiaxial cycle counting technique first proposed by Wang and Brown [10]. The Wang–Brown method identifies loading reversals based on a relative equivalent stress or strain history. By redefining the reference point and stress history for each reversal counted, this approach is able to avoid the sign problem discussed previously when computing equivalent stress ranges from non-proportional loading histories. Individual stress and strain components between the start and end points of each identified reversal can then be used to compute equivalent stress amplitudes and mean values for use in damage calculation. Implementing such an approach, however, can add a considerable amount of complexity to a fatigue life analysis.

Notched specimen fatigue life prediction adds even further complexity to the analysis procedure. In a notch analysis, depending on the notch geometry, the magnitude and location of maximum local stresses may change with a change in the nominal stress ratio. For example, the shift in location and magnitude of maximum stresses for the specimen geometry used in this study is shown in [11]. This makes fatigue life prediction for

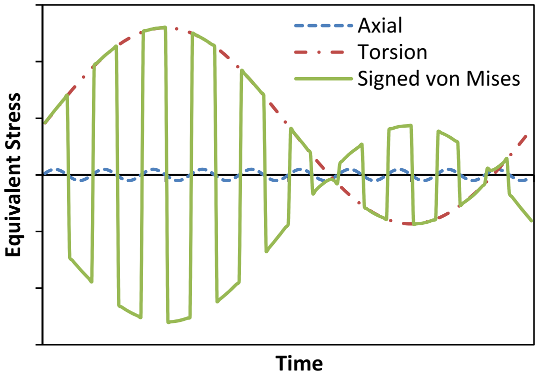

Fig. 1. Example of signed von Mises stress history under non-proportional axial–torsion loading.