International Journal of Fatigue 83 (2016) 10–23

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

Sequential Monte-Carlo sampling based on a committee of artificial neural networks for posterior state estimation and residual lifetime prediction

C. Sbarufatti *, M. Corbetta, A. Manes, M. Giglio

Politecnico di Milano, Dipartimento di Meccanica, via La Masa 1, 20156 Milan, Italy

ARTICLE INFO

Article history:
Received 29 October 2014
Received in revised form 30 April 2015
Accepted 25 May 2015
Available online 1 June 2015

Keywords:
Sequential importance resampling
Artificial neural network
Committee
Augmented state vector
Fatigue crack growth

ABSTRACT

The application of Bayesian methods to the problem of fatigue crack growth prediction has been growing in recent years. In particular, sequential Monte-Carlo sampling is often presented as an efficient model-based technique to filter the sequential measures of the damage evolution provided as an input to the algorithm. However, a lot of measures are required to reliably identify the system state condition and the underlying model parameters. Many studies rely on the availability of a relatively dense sequence of crack length measures during damage evolution, made in most cases impractical by the consequent maintenance costs. Thus, real-time damage diagnosis is a requirement to enable prognostic health management.

This work focuses on the application of sequential Monte-Carlo sampling to estimate the probabilistic residual life of a structural component subjected to fatigue crack propagation, while real-time estimation of crack length is provided through a committee of artificial neural networks, trained with finite element simulated strain patterns. Multiple crack length observations are available at each discrete time and are provided as the input to the prognostic system, based on a sequential importance resampling algorithm. Each time a new set of measures is available, the algorithm evaluates the posterior distribution of the augmented state vector, including the crack length and a material parameter governing damage evolution. This filtered information is used to numerically update the probability density functions of the residual life of the component. The methodology is applied first to a simulated crack and then to a metallic stiffened panel specimen subject to fatigue crack growth.

1. Introduction

Fatigue crack growth (FCG) is a primary degradation process in the field of metallic structure reliability, especially for aeronautical applications, and the development of accurate models to describe damage evolution is widely researched (Paris–Erdogan's law [1], Walker's law, Forman's law, NASGRO law [2], to mention a few [3]). The increase of safety requirements and the necessity of advanced maintenance strategies (e.g. the condition based maintenance approach) [4] give rise to the adoption of stochastic approaches to improve the reliability of the predictions with respect to design expectations [5]. As a matter of fact, a more precise estimate of the residual life at a specific time can be provided by the observation of the current condition of the component. This gives rise to the conditional residual life (RL) probability density function (PDF) estimate. At a specific point in time and for a particular observation of the condition of a component, a posterior PDF for the residual life (conditional on the observation) can be evaluated based on the availability of a prognostic model. This approach is particularly well described in the Bayesian updating framework [6,7], in which the a priori information on the RL is updated according to the actual observations taken by a diagnostic tool.

Recent advances on sequential Monte-Carlo methods, specifically Sequential Importance Sampling/Resampling (SIS–SIR) algorithms, allowed integrating multiple uncertainties related to the measurement system and the intrinsic randomness of the degradation phenomenon with the mathematical FCG formulations [7–9]. Their suitability for the prediction of evolving phenomena by sequentially updating of the system state estimation has already been demonstrated [8,9]. However, these algorithms require (i) a probabilistic model describing the system evolution, (ii) a stochastic measurement model relating the measure to the system state and (iii) a sufficient number of measures to guarantee the

* Corresponding author. Tel.: +39 022399 8213.
E-mail address: claudio.sbarufatti@polimi.it (C. Sbarufatti).

http://dx.doi.org/10.1016/j.ijfatigue.2015.05.017
0142-1123/

convergence of the algorithm state estimation to the target process evolution [10]. The assumption that a sufficient number of measures are available from any non-destructive testing (NDT) measuring apparatus is widely made [9–11]. This assumption is however only applicable to cases in which maintenance costs are not a primary issue and a maximum exploitation to RL estimation can only be obtained when coupled to a real-time diagnostic system providing automatic crack length measures [12], thereby entering the field of structural health monitoring (SHM) [13]. The aim of this paper is to monitor the damage evolution and to make prognosis on a relatively complex metallic structure with a riveted skin–stringer construction, where diagnosis is made by using an SHM system based on a sensor network and a committee of artificial neural networks (ANNs). This work is an initial step towards the on-line condition monitoring of aeronautical structures, including a prognostic system able to filter multiple information from the SHM diagnostic unit.

Focusing on the diagnostic problem, the algorithm for damage identification requires extensive investigations. In [12], the authors described the optimisation procedure for a diagnostic algorithm based on ANNs, trained on finite element simulated strain patterns. This algorithm is able to generalise to the experimental measures well, providing damage detection, localisation and crack length quantification. A sufficient level of generalisation is especially important for the ANN when using simulation for algorithm training. In addition to some typical regularisation techniques such as cross validation and early stopping, the authors grouped multiple ANN models trained on *Bootstrap* datasets into one committee [14], obtaining further algorithm regularisation. In practice, each time a strain pattern is provided as the input to the diagnostic algorithm, the diagnostic output is not a single indication but a series of N_{ANN} outputs, being N_{ANN} the number of ANNs belonging to the committee. Such distribution is related to the uncertainty intrinsic to the diagnostic model training procedure [14]. Various methods studying how to combine the outputs from multiple models are present in the literature [15], the simplest way consists in averaging the prediction of the set of individual models [12,15]. This combined observation can be used as the input to the prognostic algorithm. However, by simply averaging the observations the information regarding the diagnostic model uncertainty related to a particular measure is lost. A thorough method to combine the multiple outputs from a diagnostic system into the prognostic SIR framework is needed and is the objective of the present study.

In this work, at each discrete time step, the dispersion related to the entire set of multiple observations is combined with the randomness intrinsic to the FCG process, usually described within a dynamic state space (DSS) [7], and a SIR algorithm is adopted to filter the total uncertainty. This filtered information is then used to numerically update the posterior distribution of the residual life. In order to filter also the uncertainty related to the FCG model parameters [16–18], a SIR algorithm with an augmented state vector [19–22] has been implemented. One FCG model parameter typically associated with material properties in linear elastic fracture mechanics is inserted into the state vector and its PDF is sequentially updated in real time. This additional complexity is necessary due to the very high FCG variability one can expect during repeated tests [10,16]. The methodology is applied first to the identification of a simulated crack growth process and then to a real metallic stiffened panel specimen subjected to fatigue crack growth. According to the recent literature, the validation of such a SHM tool on realistic structures (i.e. the portion of a helicopter fuselage) constitutes a novelty in the aeronautics panorama.

The paper is structured as follows: the theory of SIR algorithm is shortly introduced in Section 2, with focus on the extension for diagnostic output filtering as well as for the parameter estimation with an augmented state vector. The critical aspects regarding the SIR implementation for FCG prognosis are detailed in Section 3. The results of the simulated FCG are presented in Section 4. Section 5 is dedicated to the verification of the algorithm performance during one FCG test, showing the overall prognostic system performances. A critical discussion of the work and some possible future extensions of the method are provided in the conclusive section.

2. Theory and methods

Extensive literature is available concerning the mathematical aspects of sequential Monte-Carlo filters (two remarkable examples are [6,7]), therefore this section only summarises the primary aspects of the algorithm procedure and the input requirements. In particular, the basic formulation of system state filtering by SIR algorithm is reported in Section 2.1, followed by its extension for augmented state vector filtering in Section 2.2. The algorithm modification to receive multiple observations as the input is reported in Section 2.3.

2.1. Basics on sequential importance resampling strategy

The definition of a DSS [7], including the model evolution Eq. (1) (consisting of a hidden Markov process of order one) and the observation Eq. (2) (linking the measures with the system state) is considered.

$$x_k = f(x_{k-1}, \vartheta, \omega_{k-1}) \tag{1}$$

$$z_k = h(x_k, \eta_k) \tag{2}$$

where x_k is the system state at k-th discrete time, ϑ is a vector collecting the model parameters supposed to be constant during the process evolution, ω_{k-1} is the artificial process noise and η_k is the uncertainty affecting the observation z_k. In a Bayesian updating framework, the objective is to calculate the posterior PDF of the system state at discrete time k, conditioned on the observations, indicated as $p(x_k|z_{1:k})$. This can be evaluated with the well-known prediction and updating steps, respectively performed through the Chapman–Kolmogorov equation and the Bayes' rule [7]. Nevertheless the analytical solution of this problem is only possible if the model is linear and each random process involved is Gaussian. The SIR algorithm is a recursive Bayesian filter, commonly used to approximate the state posterior distribution by a series of samples (often referred to as *particles*). At each k-th discrete time, N_s particles $x_k^{(i)}$ [$i = 1, \ldots, N_s$) approximate the posterior PDF of the system state, $p(x_k|z_{1:k})$. Each of these particles has a linked weight (3) depending on the weight at the previous time step $w_{k-1}^{(i)}$ and the likelihood of that particle given the measure. After a normalisation of the weights is performed in a way that $\sum_{i=1}^{N_S} \tilde{w}_k^{(i)} = 1$, the posterior PDF of the system state can be calculated (4). To limit sample degeneracy, particles are resampled according to the actual posterior distribution, as indicated in Eq. (5).

$$w_k^{(i)} = w_{k-1}^{(i)} p\left(z_k \middle| x_k^{(i)}\right) \tag{3}$$

$$p(x_k|z_{1:k}) \approx \sum_{i}^{N_s} \tilde{w}_k^{(i)} \delta\left(x_k - x_k^{(i)}\right) \tag{4}$$

$$x_k^{(i)} \sim p(x_k|z_{1:k}) \tag{5}$$

The procedure described through Eqs. 3–5 is repeated each time a new observation is provided by a measuring system.

Having applied the SIR algorithm to filter the state vector based on diagnostic observations, the prognosis consists in projecting the filtered particle population in time, up to the failure region that is identified for the specific use case (this may be a critical crack