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Hyper-third order full-discretization methods in milling stability prediction

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ABSTRACT

Full-discretization methods beyond the third order is not yet explored except for this work in which the fourth and fifth order methods are presented. It is seen in earlier works that accuracy of milling stability analysis using the full-discretization method rises from the first order method to the second order method and continues to rise to the third order method. It is seen in this work that the rise in accuracy of the full-discretization method with order continues to the proposed fourth order method where it (accuracy) peaks before a decline to the proposed fifth order method.

1. Introduction

Regenerative chatter is the most encountered self-excited vibration in machining operation [1,2]. It limits metal removal rate below the machine's capacity and hence reduces the productivity of the machine. Regenerative chatter is the unstable and violent tool–workpiece interaction that is caused by delay effects [3–6]. The analytical treatment of regenerative chatter as pioneered by Tobias and Fishwick [7] and Meritt [8] for turning process is the major inspiration of the wide-spread modern day analysis of cutting tool as a spring–mass–damper system subjected to cutting force reaction from a workpiece. Most work done in this direction seeks stability boundaries or lobes that separate the cutting parameter space of spindle speed and depth of cut into stable and unstable domains such that the machinist can make the most of lobeing effect by choosing the parameter combinations that give the maximum material removal rate. This is the out-of-process strategy of regenerative chatter control that is so-called because it taps the best of the system dynamics without modifying the dynamical parameters of the system [9]. The success of this approach depends on near accurate determination of modal parameters of the spring–mass–damper cutting tool model and the cutting coefficient of the adopted cutting force model. These modal parameters are normally estimated from experimental analysis that includes both stiffness and impact testing [2,10–14]. Needed cutting coefficients are normally estimated from experimental cutting force analysis that involves measuring the forces with dynamometers. Two types of experimental cutting force analysis are distinguished: *mechanics of cutting approach* [14,15] and *mechanistic approach* [10,16–18]. Numerical finite element approach to predicting machining cutting force and cutting coefficient is evolving over the past decade. This method utilizes slip line field theory [19,20]. The aspects of modeling and stability analysis of the out-of-process chatter control for turning process is relatively easy because of the fact that turning process is governed by autonomous delay differential equation (DDE) which can be completely analytically handled using the method of D-subdivision [21–24]. The autonomous nature of the DDE is due to time-invariance of the stationary chip thickness. The dynamics of regenerative chatter in milling process is much more complicated by multiplicity of cutting edges and periodically varying stationary chip thickness. Complete analytical treatment of milling process is not possible as in turning process. Just a few hybrid methods that are both analytical and numerical in nature are used in the aspect of stability analysis of the out-of-process chatter control for milling process. Some of the hybrid methods are the zeroth order approximation method [25,26], temporal finite element analysis (TFEA) [27–29], the semi-discretization method [28,30,31], the Lambert function based method [32,33], the full-discretization method [34–40], a method based on linear approximation of acceleration [41] and complete discretization [42].

2. Detailed review of the full-discretization method

The semi-discretization method [30] was earlier available for stability analysis of milling process than the full-discretization

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method [34]. The basis of their difference hinges on the extent of discretization implemented on the basic state space form of the governing DDE. For illustrative reason, we consider a basic scalar DDE model for 1 DOF milling process utilized in the two pioneering works

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh(t)}{m}\right)z(t) = \frac{wh(t)}{m}z(t-\tau) \tag{1}$$

The adjective "basic" is included to mean a milling model without the effects of some complicating phenomena like process damping, tool run-out, spindle speed variation, mode coupling, non-linearity, tools with non-uniform pitch etc. State space analysis of stability of Eq. (1) using the semi-discretization method as presented in [30] required transformation to the form

$$\dot{\boldsymbol{\xi}}(t) = \mathbf{A}(t)\boldsymbol{\xi}(t) - \mathbf{B}(t)\boldsymbol{\xi}(t-\tau) \tag{2}$$

where the periodic matrices $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are

$$\mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 - \dfrac{wh(t)}{m} & -2\xi\omega_n \end{bmatrix}, \quad \mathbf{B}(t) = \begin{bmatrix} 0 & 0 \\ -\dfrac{wh(t)}{m} & 0 \end{bmatrix} \tag{3a}$$

and the state vector $\boldsymbol{\xi}(t)$ is

$$\boldsymbol{\xi}(t) = \begin{Bmatrix} z(t) \\ \dot{z}(t) \end{Bmatrix} \tag{3b}$$

The semi-discretization of Eq. (2) was limited to the delayed term. This resulted in a series of state space ODEs paving the way for construction of system monodromy operator from which stability calculated. Versatility of the use of the semi-discretization method in analysis of more advanced milling models that include special effects like process damping [43–45], tool run-out [46], spindle speed variation [47,48], non-linearity [45], tools with non-uniform pitch [49,50] is note-worthy. By subtracting the matrix $\mathbf{B}(t)$ from matrix $\mathbf{A}(t)$ a constant matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}$ results allowing Eq. (2) to become

$$\dot{\boldsymbol{\xi}}(t) = \mathbf{A}\boldsymbol{\xi}(t) + \mathbf{B}(t)\boldsymbol{\xi}(t) - \mathbf{B}(t)\boldsymbol{\xi}(t-\tau) \tag{4}$$

The milling model having the form of Eq. (4) was given extended discretization in the pioneering full-discretization method [34] to include the delayed term $\mathbf{B}(t)\boldsymbol{\xi}(t-\tau)$ and current term $\mathbf{B}(t)\boldsymbol{\xi}(t)$. The principal period which is same as the discrete delay τ is discretized into equal time intervals Δt. The discrete interval is given as $\Delta t = \tau/k = t_{i+1} - t_i$ where $i = 0, 1, 2, \ldots, (k-1)$, $t_i = i\tau/k = i\Delta t = i(t_{i+1} - t_i)$. The discretization integer k is called the approximation parameter. Eq. (4) is represented as

$$\dot{\mathbf{y}}(t) = \mathbf{A}\mathbf{y}(t) + \mathbf{B}(t)\mathbf{y}(t) - \mathbf{B}(t)\mathbf{y}(t-\tau) \tag{5}$$

in the discrete interval $[t_i, t_{i+1}]$ and solved in [34] using direct integration scheme to give

$$\mathbf{y}_{i+1} = \mathrm{e}^{\mathbf{A}\Delta t}\mathbf{y}_i + \int_{t_i}^{t_{i+1}} e^{\mathbf{A}(t_{i+1}-s)}[\mathbf{B}(s)\mathbf{y}(s) - \mathbf{B}(s)\mathbf{y}(s-\tau)]\mathrm{d}s \tag{6}$$

As commented in [34], the works [51,52] were inspirational to the advent of the full-discretization method in milling stability analysis. Searching the keyword "full-discretization method" on Science direct and Google shows that the name "full-discretization method" has selectively been used to refer to methods in which the integration scheme is solved after the terms $\mathbf{B}(s)\mathbf{y}(s)$ and $\mathbf{B}(s)\mathbf{y}(s-\tau)$ are replaced with Lagrange [34,36], Newton [38] or Hermite [39] interpolation polynomial. The view in this work is that any method based on Eqs. (5) and (6) belongs to the full-discretization method irrespective of how the integration scheme is handled. The reason for this view being that the name stems from the method and extent of discretization and not on the method of solution. This means that other methods which are not explicitly called "full-discretization method" but based on Eqs. (4)–(6) are full-discretization methods. For example the method in [37,53] in which the integration scheme in Eq. (6) is handled by the method of numerical integration belongs to the full-discretization method. Also based on this criterion, the spectral method [54] for milling stability analysis belongs to the full-discretization method. The method in the work [55] called "improved semi-discretization method" in which milling process with spindle speed variation is analyzed belongs to the Full-discretization method. The methods in [40] which utilize least squares approximation of the terms $\mathbf{B}(s)\mathbf{y}(s)$ and $\mathbf{B}(s)\mathbf{y}(s-\tau)$ belongs to the full-discretization method. It has been found that the full-discretization methods that are based on interpolation theory and those that are based on approximation theory (least squares method) are of same accuracy [40]. The latter are superior for producing monodromy matrices that are generated with less number of numerical calculations thus saving computational time. This is the reason for which proposals for higher order full-discretization methods in this work will be based on least squares method. Accuracy of the full-discretization method improves with increase of order of interpolation/approximation of state term from one to three [36,38,40]. Against the backdrop of the work [56] the trend of increasing accuracy with rise in order of interpolation/approximation from one to three is not guaranteed beyond the third order theory. Even in the face of this curiosity no effort is yet seen to go beyond third order interpolation/approximation theory in the full-discretization method probably due to heavy analytical involvement. Fourth and fifth order cases of full-discretization method are considered here in order for the first time settle this curiosity of what becomes of accuracy of the full-discretization method beyond third order theory. This as stated earlier is done from the perspective of approximation theory (least squares approximation) where it should be understood that results pertaining to accuracy are also applicable if interpolation theory were used.

3. Mathematical model of the 2DOF milling process

The matrix delay differential equation governing the regenerative motion of a 2DOF milling tool is

$$\ddot{\mathbf{z}}(t) + \mathbf{M}^{-1}C\dot{\mathbf{z}}(t) + \mathbf{M}^{-1}\mathbf{K}\mathbf{z}(t) = \mathbf{M}^{-1}\mathbf{H}[\mathbf{z}(t) - \mathbf{z}(t-\tau)] \tag{7}$$

where $\mathbf{z}(t) = \left\{z_x(t) \quad z_y(t)\right\}^T$ is the vector of regenerative motion in the feed and feed-normal directions respectively and

$$\mathbf{M} = \begin{bmatrix} m_x & 0 \\ 0 & m_y \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_x & 0 \\ 0 & c_y \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} -wh_{xx}(t) & -wh_{xy}(t) \\ -wh_{yx}(t) & -wh_{yy}(t) \end{bmatrix}. \tag{8}$$

The coupling in the model which stems from the two dimensional feed of the force law is contained in the τ-periodic matrix $\mathbf{H}$ because of presence of non-zero off diagonal elements. The non-linear cutting force law for the 2DOF system is used in the form

$$F_{t,j}(t) = C_t w\left[f_{a,x}\sin\theta_j(t) + f_{a,y}\cos\theta_j(t)\right]^{\gamma} \tag{9a}$$

$$F_{n,j}(t) = \mathcal{X}F_{t,j}(t) \tag{9b}$$

where w is depth of cut, C_t and C_n are the tangential and normal cutting coefficients associated with the workpiece material properties and tool shape, $\mathcal{X}$ is the ratio C_n/C_t and $f_{a,x}$ and $f_{a,y}$ are the respective feeds in the feed and feed-normal directions under regenerative effects. A modeling procedure for 2DOF milling