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# Verification of fatigue critical plane position according to variance and damage accumulation methods under multiaxial loading

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ABSTRACT

The paper presents a comparison of fracture plane position gained from experimental tests of specimens under multiaxial loading and theoretical ones from calculation according to variance and damage accumulation methods. In the variance method it is assumed that the plane in which the maximum variance of the equivalent stress appears is critical for a material and the fatigue fracture should be expected in this plane. In the damage accumulation method the fatigue critical plane is assumed to be the plane which suffered the greatest damage during service loading. For both methods the equivalent stress is calculated according to the multiaxial fatigue failure criteria of (i) maximum normal stresses, (ii) maximum shear stresses as well as (iii) maximum normal and shear stresses in the critical plane.



## 1. Introduction

The critical plane position under multiaxial random loading can be determined by means of one of the following methods:

- The method of variance.
- The method of damage accumulation.
- The method of weight functions.

The method of variance [1–3] relies on searching the maximum variance of equivalent stress (or strain or energy parameter), according to the selected fatigue failure criterion. The plane where the variance reaches its maximum is assumed to be the critical plane.

The method of damage accumulation [4–8] seems to be the most interesting due to its close relation to the idea of critical plane. In this method, the selected fatigue failure criterion is applied for searching the maximum damage plane, i.e. the plane of the minimum fatigue life. This is an iterative method, so that the search of the critical plane requires multiple repetitions of all the calculation algorithm [9,10].

The method of weight functions [11–13] relies on searching averaged positions of directions of principal stress axes by suitable weight functions. The critical plane position is determined in relation to those axes, taking into account the kind of the material (elastic–plastic, elastic-brittle, or intermediate). Under multiaxial service loadings and the fatigue failure criteria based on the critical plane, the algorithm presented in Fig. 1 is usually applied. The algorithm is used for calculations of the fatigue life of materials under random loadings with zero mean values [6,14].

The aim of the paper is to compare of the variance method and the method of damage accumulation for determination of the critical plane position with the results of experiments.

## 2. Experimental procedure

### 2.1. Material and specimen

The material under investigation is the construction steel 10HNAP (S355J2G1W). Characteristics of this material are given by the Polish-European Standard (PN-EN 10155) concerning low-alloyed and corrosion-resisting steel. The steel 10HNAP is a weldable general purpose steel. After the analysis of axial cyclic stress–strain curves, it was concluded that 10HNAP steel is a cyclically stable material during the fatigue tests. Table 1 contains the chemical composition of the material, whereas some of its monotonic quasi-static tension and fatigue properties are included in Table 2. Specimens of round sections were tested (Fig. 2), cut from sheets 16 mm thick, along the rolling direction. Two diameters of the specimens were considered: $d = 7.5$ mm for variable amplitude loading, and $d = 8$ mm for pseudo-random loading. The microstructure of the material strongly influences the crack paths. In elastic–plastic materials the grain size is one of the most important factors deciding about critical plane position; the smaller diameter of the

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**Nomenclature**

| | |
|---|---|
| $A$ | coefficient allowing to include amplitudes below the fatigue limit in the process of fatigue damage accumulation ($a = 0.5$ is assumed in this paper) |
| $d$ | diameter of specimen |
| $f_s$ | sampling frequency |
| $k$ | number of intervals of the amplitude histogram |
| $m$ | coefficient of Wöhler's curve slope |
| $n_i$ | number of cycles with amplitude $\sigma_{eq,ai}$ |
| $r_{\sigma\tau}$ | cross-correlation coefficient between normal stresses and shear stresses |
| $E$ | Young's modulus |
| $I$ | irregularity factor of loading |
| $M^*$ | a number of signal peaks in a time unit |
| $M_B$, $M_T$ | bending moment and torsional moment, respectively |
| $N^*$ | a number of excesses above the zero level with a positive slope in a time unit |
| $N_0$ | number of cycles corresponding to the fatigue limit $\sigma_{af}$ |
| $S(T_O)$ | fatigue damage degree at the observation time $T_O$ for stress $\sigma_{eq}(t)$ |
| $T_O$ | observation time |
| $T_{cal}$, $T_{exp}$ | calculated and experimental fatigue life, respectively |
| $W_B$, $W_T$ | section modulus for bending and torsion, respectively |
| $\alpha$ | angle determining the critical plane position |
| $\alpha_{FP}$ | average angle of fracture plane position |
| $\lambda_\sigma$ | ratios of maximum stresses |
| $\mu_\sigma = \mu_{x1}$ | variance of normal stress $\sigma_x$ |
| $\mu_\tau = \mu_{x2}$ | variance of shear stress $\tau_{xy}$ |
| $\mu_{\sigma\tau} = \mu_{x1x2}$ | covariance of normal $\sigma_x$ and shear $\tau_{xy}$ stresses |
| $\mu_{\sigma eq}$ | variance of equivalent stress $\sigma_{eq}$ |
| $\sqrt{\mu_\sigma}$, $\sqrt{\mu_\tau}$ | standard deviation of normal and shear stresses, respectively |
| $\nu$ | Poisson's ratio |
| $\sigma_{af}$ | fatigue limit under fully reversed bending |
| $\sigma_{eq,ai}$ | amplitude of the equivalent stress |
| $\sigma_x(t)$ | normal stress along the specimen axis |
| $\sigma_Y$, $\sigma_U$ | yield strength and ultimate strength in quasi-static tension, respectively |
| $\tau_{af}$ | fatigue limit under fully reversed torsion |
| $\tau_{xy}(t)$ | shear stress in the specimen cross section |

grain is, the smaller differences between directions for stages I and II can be observed. Additional impact on the difference of critical plane position, related to the grain, have non-metallic inclusions. In Fig. 3 an exemplary stage I for the crack initiation and stage II for the crack propagation is shown. In Fig. 3 significant differences between the initiation plane and the propagation plane can be noticed.

Moreover, the kind of microstructure may have a considerable influence on fatigue fracture plane characteristic in material. Influence of microstructure on crack path growth, hence on change of the crack plane position, was shown in Fig. 4. In Fig. 4 two kinds of materials are presented (10HNAP steel and Ti–6Al–4V titanium alloy), which allow to demonstrate a different growth of the crack paths (stage II). For 10HNAP (Fig. 4a) the main crack does not change its direction, whereas for Ti–6Al–4V alloy (Fig. 4b) multiple changes of the main direction can be seen, which is crossed by many secondary cracks not occurring in 10HNAP steel. In both cases, the cracks shown in Fig. 4 started to propagate from the left side of this figure. In Fig. 4 significant differences in the grain size are noticed; for 10HNAP steel an average grain size is 22.5 μm while for Ti–6Al–4V the structure is characterized by the lengthy grains of phase α with length up to 50 μm.

Metallographic tests of the steel show a fine grained ferritic-pearlitic structure (Fig. 5) with average diameter of the grain equal 22.5 μm [15].

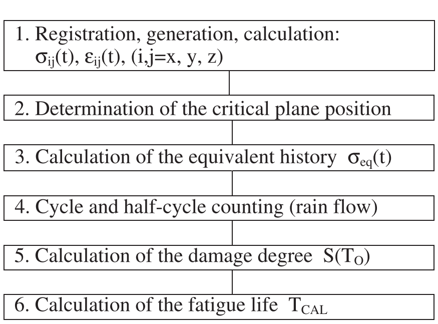


**Fig. 1.** Algorithm for determination of the fatigue life of materials under multiaxial service loading.

*2.2. Fatigue testing*

The tests were performed in the high cycle fatigue regime (HCF) under variable amplitude and pseudo-random bending with torsion loading at Opole University of Technology. Two fatigue stands were used to carry out fatigue tests [5,16]. The first stand MZGS-200L [5] includes two electromagnetic actuators and it is able to work under variable amplitude non-proportional bending with torsion loading. One of the actuators forces bending, and the other one torsion of the specimen. The stand MZGS-200L allows to test the influence of the cross-correlation coefficient $r_{\sigma\tau} = \frac{\mu_{\sigma\tau}}{\sqrt{\mu_\sigma \cdot \mu_\tau}}$ between the normal and shear stresses on fatigue life. Histories of bending and torsion moments were independently generated. The loadings for which $r_{\sigma T} = \pm 1$ are assumed to be proportional, in all the other cases loadings are non-proportional. The fatigue test stand includes a computer, generating electric signals $x(t)$ and $y(t)$ transmitted to amplifiers. The intensified signals from the amplifiers are sent to electromagnetic actuators. Each inductor core is joined with a lever. One lever is used for applying the bending moment $M_B(t)$ to the specimen, generating nominal normal stress $\sigma(t) = M_B(t)/W_B$. The other lever is used for applying the torsional moment $M_T(t)$ generating nominal shear stress $T(t) = M_T(t)/W_T$, where $W_B$ and $W_T$ are section modulus for bending and torsion, respectively. The tests were carried out under variable amplitude loading with the frequency of 20 Hz and the coefficient of irregularity $I = 1$ ($I = N^*/M^*$, where $N^*$ = number of crosses the zero level with a positive slope in a time unit, $M^*$ = number of signal peaks in a time unit).

The second stand MZGS-200PL (pseudo-random loading)[16], with the dominating frequency of 28.8 Hz for bending and 30 Hz for torsion, is applied for fatigue tests of specimens made of structural materials subjected to non-proportional combinations of the bending moment $M_B(t)$ and the torsional moment $M_T(t)$. Histories of these moments are independent (separate drives and control

**Table 1**
Chemical composition of 10HNAP steel (wt%).

| Material | C | Mn | Si | P | S | Cr | Ni | Cu | Fe |
|---|---|---|---|---|---|---|---|---|---|
| 10HNAP | 0.11 | 0.52 | 0.26 | 0.098 | 0.016 | 0.65 | 0.35 | 0.26 | Bal. |