International Journal of Fatigue 42 (2012) 183–193

Contents lists available at SciVerse ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

A novel methodology to predict the endurance domain for a material and its evolution using a generalized fracture mechanics framework

Raùl de Moura Pinho [a,b], Sylvie Pommier [a,*], Caroline Mary [b], Arnaud Longuet [b], François Vogel [c]

[a] LMT-Cachan, ENS Cachan/CNRS/UPMC/UniverSud Paris, 61, Avenue du Président Wilson, 94235 Cachan, France
[b] Snecma-Safran, Rond-point René Ravaud, Réau, 77550 Moissy-Cramayel, France
[c] Turbomeca-Safran, F-64511 Bordes, France

ARTICLE INFO

Article history:
Received 22 October 2010
Received in revised form 27 July 2011
Accepted 28 October 2011
Available online 9 November 2011

Keywords:
Endurance criterion
LCF/HCF
Crack tip plasticity
Fatigue
Short crack

ABSTRACT

Materials in rotating machinery are typically subjected to loading conditions that combine both low cycle fatigue (LCF) and high cycle vibratory fatigue (HCF). In operating conditions, the number of allowable LCF cycles may be limited, but under no circumstances, superimposed HCF vibratory cycles can be allowed to produce fatigue damage. This paper aims at proposing an approach to account for the effect of variable amplitude LCF cycles on the endurance domain relative to HCF cycles. For this purpose, a LEFM-based approach is proposed. The material is assumed to contain flaws, and their growth is assumed to stem from crack tip plasticity. As a consequence of these "classical" assumptions, the endurance domain of the material can alternatively be viewed as an elastic domain for the crack tip region. A generalized von Mises yield criterion is hence introduced to model that elastic domain. For this purpose, rather than considering the distortional elastic energy density (von Mises), the Westergaard's stress field is used to calculate the distortional elastic energy within a distance δ from the crack tip. A non-local yield criterion for the crack tip region is obtained. The first non-singular terms (e.g., the T-stresses) are included in the Westergaard's stress functions to make it possible to use it also for mechanically short cracks. Then to account for the effect of LCF cycles, the elastic domain is allowed to evolve when crack tip plasticity occurs. The center of the endurance domain is defined as an internal variable that stands for the internal stress field within the crack tip region due to constrained plastic deformation. As long as the loading path remains inside the elastic domain, no plasticity, and hence no crack growth, is expected. If the yield surface is reached, plastic strain occurs and the elastic domain is displaced. In this study, elastic–plastic finite element computations are used to determine whether the crack tip region behaves elastically or plastically. For this purpose, the velocity field within the crack tip region is partitioned into elastic and plastic parts, each part being approached as the product of an intensity factor and a spatial reference field. The plastic intensity factor is then used as a global measure of the plasticity rate within the crack tip region. In this paper, the approach was applied to a forged Ti–6Al–4V titanium alloy, used in compressor blades of gas turbine engines.

1. Introduction

Compressor aerofoils in gas turbine engines are subjected to the combination of aeroelastic vibrations (HCF) and to low frequency loading histories (LCF) due to ambient temperature variations and centrifugal forces changes during maneuvers. In operating conditions, complex variable amplitude LCF loading histories may be encountered and may be allowed to produce limited fatigue damage, but under no circumstances, HCF vibratory cycles can be allowed to produce fatigue damage. The aim of this paper was to propose a novel methodology to account for the effect of complex variable amplitude LCF cycles on the endurance domain of a material relative to HCF vibratory cycles.

The material considered in this paper is a Ti–6Al–4V titanium alloy used in forged compressor aerofoils. For constant amplitude uniaxial loading conditions, the allowable vibratory stress amplitude at 10^6 cycles was determined, in alternated tension and torsion at $R = -1$ by Delahay [1] and in alternated tension at various stress ratios ($R = -1$, 0, 0.5, and 0.8) by Lanning et al. [2]. Specific fatigue tests were also performed by Nowell et al. [3] to determine the detrimental effect of superimposed LCF cycles or that of defects (due to foreign object damage for instance).

As the cost of such experimental campaign becomes prohibitive when the loading complexity increases, fatigue criteria are required to predict the fatigue resistance in complex cases (e.g., multiaxial, variable amplitude) using experimental data obtained by

* Corresponding author. Tel.: +33 1 47 40 28 69; fax: +33 1 47 40 22 40.
E-mail address: pommier@lmt.ens-cachan.fr (S. Pommier).

0142-1123/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijfatigue.2011.10.011

Nomenclature

LEFM	linear elastic fracture mechanic
CTOD	crack tip opening displacement
DF	discontinuous field across the crack faces
CF	continuous field across the crack faces
K_I^∞	nominal applied stress intensity factor
T^∞	nominal applied *T*-stress
T_z^∞	nominal applied T_z-stress
$\underline{v}(\underline{x}, t)$	velocity of the point $\underline{x}$ in the crack tip region
$\underline{u}_I^e, \underline{u}_I^c$	elastic and plastic reference spatial DF
$\underline{u}_T^e, \underline{u}_T^c$	elastic and plastic reference spatial CF
$d\tilde{K}_I/dt, d\rho_I/dt$	intensity factor rates of $\underline{u}_I^e$ and $\underline{u}_I^p$
$d\tilde{T}/dt, d\rho_T/dt$	intensity factor rates of $\underline{u}_T^e$ and $\underline{u}_T^c$

conventional tests (uniaxial, constant amplitude). Several criteria are available, among which most are based on local mechanical quantities (point method) [4–12]. As these approaches usually fail to account for the notch effect in fatigue [13–16], non-local criteria have also been developed [17–19]. Yet, the results given are meaningful only if the load is periodic. Incremental approaches introducing a cumulative damage parameter were then developed to account for non-periodical load schemes (e.g., [20]).

However, these approaches were all developed within the framework of continuum mechanics, which makes difficult to account for the effects of short cracks and defects [21–26]. The fatigue response of a notched component, for instance, is known to depend on the stress gradient (crack arrest phenomena) as well as on the feature size (effective volume of stressed material in relation with the probability of a crack initiation site).

The framework of fracture mechanics is better suited to account for defects and short cracks. Traditional approaches are based on the Kitagawa and Takahashi diagram [21] and the El Haddad equation [22]. However, the use of such approaches in multiaxial and variable amplitude loading conditions requires further developments. Endo and McEvily [25,26], for instance, proposed a modified linear elastic fracture mechanics analysis for the evaluation of the threshold behavior of small cracks under complex loading conditions.

In this paper, a LEFM-based approach is proposed. The material is assumed to contain flaws, and crack growth is assumed to stem from crack tip plasticity. According to these assumptions, the endurance domain of the material can alternatively be viewed as an elastic domain for the crack tip region. A generalized von Mises yield criterion was thus introduced [27,28]. To construct that criterion, rather than considering the local stress tensor and the distortional elastic energy density (von Mises), the Westergaard's stress field is used. The distortional elastic energy in this field per unit length of the crack front and within a distance δ from the crack tip is determined. The result is a non-local generalized von Mises criterion for the crack tip region. The first non-singular terms (e.g., the *T*-stresses) are considered in the Westergaard's stress field to make it possible to use it also for mechanically short cracks [27,28]. Section 2 of this paper explains how this criterion was established and how it can be used. By way of illustration, the criterion is identified using data "for" the Ti–6Al–4V alloy from the literature [1,2,29].

Then to account for the effect of a complex history (LCF cycles), the endurance domain is allowed to evolve when plastic deformation occurs within the crack tip region. The center of the endurance domain (elastic domain) is defined as an internal variable that stands for the internal stress field inherited from constrained crack tip plasticity in small-scale yielding conditions. As long as the loading path remains inside the elastic domain, no plasticity, and hence no crack growth, is expected. If the yield surface is reached, plastic strain occurs and the elastic domain is displaced. The evolution of the center of the elastic domain is expected to allow predicting the effect of load events producing limited plasticity on the endurance domain of the material.

Elastic–plastic finite element computations are used to determine whether the crack tip region behaves elastically or plastically after it was subjected to a complex loading history, and to get the displacement of the center of the endurance domain induced by that history (Section 3 of this paper). The methodology is adapted from that developed for long cracks in mixed mode conditions by Pommier et al. [28,30–32].

The approach was applied to a forged Ti–6Al–4V titanium alloy, used in compressor blades of gas turbine engines to predict the fatigue strength at 10^6 cycles in constant amplitude fatigue for various mean stresses and in combined LCF/HCF fatigue (Section 4).

2. Generalized von Mises yield criterion

2.1. Assumptions

The criterion used in this paper is a generalized von Mises yield criterion for the crack tip region introduced by Thieulot-Laure et al. [27,33]. The material is assumed to contain small cracks. We consider either a statistical distribution of cracks sizes (if known) or an equivalent flaw size. The criterion is based on the assumption that fatigue cracks propagate if cyclic plastic strain is experienced at crack tip. The criterion is thus expressed as a threshold for plastic yield for a region of material located within a distance δ to the crack tip and per unit of length of the crack front. This radius δ is a length scale parameter to be identified from experiments. Like the empirical equation of El Haddad et al. [22], the criterion of Thieulot-Laure et al. [27] requires the knowledge of three material parameters:

- the actual flaw size or an equivalent flaw size (a_o),
- the non-propagation threshold for long cracks ΔK_{th},
- a length scale parameter (δ in this criterion [27,33]).

The evolution of the fatigue strength with the defect size, as shown by Kitagawa and Takahashi [21], is just as well reproduced by the Thieulot-Laure's criterion or by the El Haddad's equation [22]. However, the Thieulot-Laure's criterion is a plastic yield threshold that makes it possible to use it for either uniaxial or multiaxial loading conditions. Examples are given below. It will also be shown in Section 3 that it can be extended to account for the effect of complex LCF histories.

The yield criterion for the crack tip region is obtained as follows. The material is assumed to obey the von Mises yield criterion at the local scale. The von Mises criterion is a critical distortional elastic energy density criterion. In order to calculate the distortional elastic energy density within the crack tip region [27,30,33], the stress, strain, and displacement fields at crack tip from LEFM are used. In addition, since it is aimed at using this criterion for small cracks, higher-order terms in the asymptotic LEFM development are also considered. The first-order non-singular terms (*T*-stresses) were therefore included in this approach.