International Journal of Fatigue 36 (2012) 109–119

Contents lists available at SciVerse ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

The numerical simulation of fatigue crack growth using extended finite element method

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ARTICLE INFO

Article history:
Received 25 March 2011
Received in revised form 11 August 2011
Accepted 16 August 2011
Available online 31 August 2011

Keywords:
Fatigue life
Cracks
Holes
Inclusions
SIFs

ABSTRACT

In the present work, the fatigue life of homogeneous plate containing multiple discontinuities (holes, minor cracks and inclusions) is evaluated by extended finite element method (XFEM) under cyclic loading condition. The multiple discontinuities of arbitrary size are randomly distributed in the plate. The values of stress intensity factors (SIFs) are extracted from the XFEM solution by domain based interaction integral approach. Standard Paris fatigue crack growth law is used for the life estimation of various model problems. The effect of the minor cracks, voids and inclusions on the fatigue life of the material is discussed in detail.

1. Introduction

The analysis of fatigue crack growth is very important to ensure the reliability of structures under cyclic loading conditions. The fatigue life of components is mainly predicted by traditional strength based theories. These theories do not account the presence of defects, irregularities and discontinuities, which are either developed at the manufacturing stage or during application. Therefore, the fracture based numerical simulations have extensive application to quantify and predict the fatigue life of component in presence of such defects and discontinuities. An accurate evaluation of fracture parameters such as stress intensity factors (SIFs) becomes quite essential for the simulation based life cycle design analysis. To simulate cracked structures, a number of methods such as boundary element method [1–3], meshfree methods [4–7], finite element method (FEM), and finite difference method (FDM) are available. FEM has been in the forefront of numerical methods used for the simulation of fatigue fracture problems. A number of approaches have been developed in FEM over the years, which makes it as a most suited method for analyzing the asymptotic stress fields at the crack tip. However, FEM requires that the crack surface should coincide with the edge of the finite elements, i.e. a conformal mesh is needed besides special elements to handle crack tip asymptotic stresses. Hence, the modeling and simulation of several discontinuities and defects using FEM becomes quite cumbersome. To overcome these difficulties, a novel approach known as extended finite element method (XFEM) [8–12] has been developed. This method allows the modeling of the crack geometry independent of the mesh, and completely avoids the need of re-meshing as the crack grows. In this method, the modeling of a crack growth and arbitrary discontinuities is performed by enriching the approximation function [9]. The level set method [12–16] has been widely used to model the crack growth. Till date, XFEM has extensively used to solve the problems of fracture mechanics including crack growth with frictional contact [17], cohesive crack propagation [18–21], quasi-static crack growth [22], arbitrary branched and intersecting cracks [23], and cracks in shells [24], fatigue crack propagation [25–28], stationary and growing cracks [15], and three-dimensional crack propagation [29,30].

The main aim of this paper is to accurately evaluate the fatigue life of structures/components having multiple discontinuities such as holes, cracks and inclusions. The plane crack problems are chosen and solved by XFEM in the presence of multiple arbitrary sizes and randomly located discontinuities. The fatigue crack propagation has been simulated by treating the crack growth as a linear combination of line segments. The direction of crack growth is obtained by maximum principal stress criterion [7]. The stress intensity factors (SIFs) are extracted from the XFEM solution by domain based interaction integral approach. After evaluating SIFs at the crack tip, Generalized Paris' law is used to compute the fatigue life. A comparison of XFEM results with those obtained by re-meshing approach (ANSYS) and experimental results has also been presented [31–35] for few model problems.

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doi:10.1016/j.ijfatigue.2011.08.010

2. Numerical formulation

2.1. Governing equations

A given domain (Ω) boundary is partitioned into displacement (Γ_u), traction (Γ_t) and traction-free (Γ_c) boundaries as shown in Fig. 1. The equilibrium conditions and boundary conditions [10] are given as

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \quad \text{in} \quad \Omega \tag{1}$$

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{n}} = \bar{\mathbf{t}} \quad \text{on} \quad \Gamma_t \tag{1a}$$

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{n}} = \mathbf{0} \quad \text{on} \quad \Gamma_c \tag{1b}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{u}$ is the displacement field, $\mathbf{b}$ is the body force per unit volume and $\hat{\mathbf{n}}$ is the unit outward normal. For small displacements, the kinematics equations consist of the following strain–displacement relations:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\mathbf{u}) = \nabla_s \mathbf{u} \tag{2}$$

where $\nabla_{\mathbf{s}}$ is the symmetric part of the gradient operator. The boundary conditions

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on} \quad \boldsymbol{\Gamma}_{\mathbf{u}} \tag{3}$$

The constitutive relation for the elastic material under consideration is given by Hook's law

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} \tag{4}$$

where $\mathbf{D}$ is the Hooke's tensor.

2.2. Weak formulation

A weak form of the equilibrium equation [9] can be written as

$$\int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{v}) d\Omega = \int_{\Omega} \mathbf{b} \cdot \mathbf{v} \, d\Omega + \int_{\Gamma_t} \bar{\mathbf{t}} \cdot \mathbf{v} \, d\Gamma \tag{5}$$

By substituting the trial and test functions in above equation and using the arbitrariness of the nodal variations, the following discrete equations are obtained

$$[\mathbf{K}]\{\mathbf{d}\} = \{\mathbf{f}\} \tag{6}$$

where $\mathbf{d}$ is the vector of nodal unknowns, $\mathbf{K}$ and $\mathbf{f}$ are the global stiffness matrix and external force vector respectively.

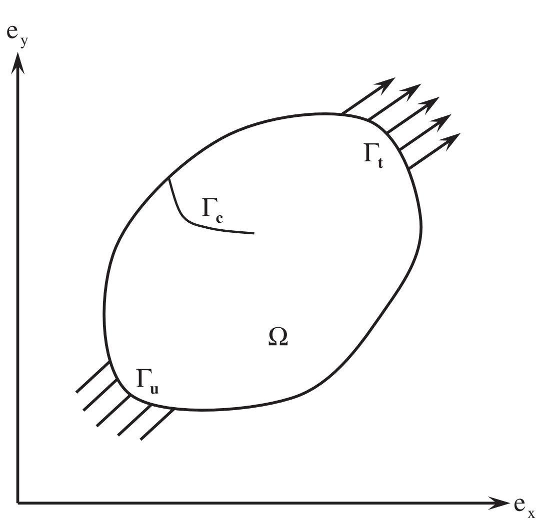

Fig. 1. Domain with a discontinuity (crack).

2.3. XFEM approximation for cracks

In two-dimensional crack modeling, the enriched trial and test functions [10,16,36] are written in general form as

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i=1}^{n} N_i(\mathbf{x}) \left[\bar{\mathbf{u}}_i + \underbrace{[\mathrm{H}(\mathbf{x}) - H(\mathbf{x}_i)]\mathbf{a}_i}_{i \in n_r} + \underbrace{\sum_{\alpha=1}^{4} [\beta_\alpha(\mathbf{x}) - \beta_\alpha(\mathbf{x}_i)]\mathbf{b}_i^\alpha}_{i \in n_A} \right] \tag{7}$$

where $\bar{\mathbf{u}}_i$ is a nodal displacement vector associated with the continuous part of the finite element solution. n is the set of all nodes in the mesh, n_r is the set of nodes belonging to those elements which are completely cut by the crack, n_A is the set of nodes belonging to those elements, which are partially cut by the crack. $\mathbf{a}_i$ is the nodal enriched degree of freedom associated with Heaviside function $H(\mathbf{x})$ (Heaviside is +1 on one side of the discontinuity and −1 on other side of the discontinuity), and $\mathbf{b}_i^\alpha$ are the nodal enriched degree of freedom associated with crack tip enrichment, $\beta_\alpha(\mathbf{x})$. The crack tip enrichment are defined as

$$\beta_\alpha(\mathbf{x}) = \left[\sqrt{r}\cos\tfrac{\theta}{2}, \quad \sqrt{r}\sin\tfrac{\theta}{2}, \quad \sqrt{r}\cos\tfrac{\theta}{2}\sin\theta, \quad \sqrt{r}\sin\tfrac{\theta}{2}\sin\theta \right] \tag{8}$$

where r and θ are the local crack tip parameters.

2.4. XFEM approximation for holes

The XFEM approximation for holes [16] can be written as

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i=1}^{n} N_i(\mathbf{x}) \left[\bar{\mathbf{u}}_i + \underbrace{[\mathrm{H}(\mathbf{x}) - H(\mathbf{x}_i)]\mathbf{a}_i}_{i \in n_r} \right] \tag{9}$$

The Heaviside jump function, $\mathrm{H}(\mathbf{x})$ takes a value of +1 on one side and 0 on the other side i.e. inside the hole.

2.5. XFEM approximation for inclusions

The XFEM approximation for inclusions [16] can be written as

$$\mathbf{u}^h = \sum_{j=1}^{n} N_j(\mathbf{x})[\mathbf{u}_j + \chi[\psi(\mathbf{x})]a_j] \tag{10}$$

where $\chi[\psi(\mathbf{x})]$ is a local enrichment function, defined as $\chi[\psi(\mathbf{x})] = |\psi(\mathbf{x})|$ and $\psi(\mathbf{x})$ is the level set function.

2.6. XFEM formulation for a crack

Using the approximation function defined in Eq. (7) for a crack, the elemental matrices, $\mathbf{K}$ and $\mathbf{f}$ are obtained as

$$\mathbf{K}_{ij}^e = \begin{bmatrix} K_{ij}^{uu} & K_{ij}^{ua} & K_{ij}^{ub} \\ K_{ij}^{au} & K_{ij}^{aa} & K_{ij}^{ab} \\ K_{ij}^{bu} & K_{ij}^{ba} & K_{ij}^{bb} \end{bmatrix} \tag{11a}$$

$$\mathbf{f}^h = \left\{ \mathbf{f}_\mathbf{i}^\mathbf{u} \quad \mathbf{f}_\mathbf{i}^\mathbf{a} \quad \mathbf{f}_\mathbf{i}^\mathbf{b1} \quad \mathbf{f}_\mathbf{i}^\mathbf{b2} \quad \mathbf{f}_\mathbf{i}^\mathbf{b3} \quad \mathbf{f}_\mathbf{i}^\mathbf{b4} \right\}^T \tag{11b}$$

The sub-matrices and vectors that appear in the foregoing equations are given as

$$\mathbf{K}_{ij}^{rs} = \int_{\Omega^e} (\mathrm{B}_i^r)^T \mathbf{D} \mathrm{B}_j^s h \, d\Omega \quad \text{where} \quad r, s = u, a, b \tag{12}$$

$$\mathbf{f}_i^u = \int_{\Omega^e} N_i \mathbf{b} \, d\Omega + \int_{\Gamma_t} \mathrm{N}_i \bar{\mathbf{t}} \, d\Gamma \tag{13}$$

$$\mathbf{f}_i^a = \int_{\Omega^e} N_i(H(\mathbf{x}) - H(\mathbf{x}_i))\mathbf{b} \, d\Omega + \int_{\Gamma_t} N_i(H(\mathbf{x}) - H(\mathbf{x}_i))\bar{\mathbf{t}} \, d\Gamma \tag{14}$$