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Analytical modeling of spindle–tool dynamics on machine tools using Timoshenko beam model and receptance coupling for the prediction of tool point FRF

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Abstract

Regenerative chatter is a well-known machining problem that results in unstable cutting process, poor surface quality and reduced material removal rate. This undesired self-excited vibration problem is one of the main obstacles in utilizing the total capacity of a machine tool in production. In order to obtain a chatter-free process on a machining center, stability diagrams can be used. Numerically or analytically, constructing the stability lobe diagram for a certain spindle–holder–tool combination implies knowing the system dynamics at the tool tip; i.e., the point frequency response function (FRF) that relates the dynamic displacement and force at that point. This study presents an analytical method that uses Timoshenko beam theory for calculating the tool point FRF of a given combination by using the receptance coupling and structural modification methods. The objective of the study is two fold. Firstly, it is aimed to develop a reliable mathematical model to predict tool point FRF in a machining center so that chatter stability analysis can be done, and secondly to make use of this model in studying the effects of individual bearing and contact parameters on tool point FRF so that better approaches can be found in predicting contact parameters from experimental measurements. The model can also be used to study the effects of several spindle, holder and tool parameters on chatter stability. In this paper, the mathematical model, as well as the details of obtaining the system component (spindle, holder and tool) dynamics and coupling them to obtain the tool point FRF are given. The model suggested is verified by comparing the natural frequencies of an example spindle–holder–tool assembly obtained from the model with those obtained from a finite element software.

Keywords: Chatter stability; Machine tool dynamics; High-performance machining

1. Introduction

Self-excited chatter vibrations caused by regeneration of waviness result in reduced productivity and poor surface finish in machining operations. It is well known that the regeneration effect is related to the phase between two vibration waves during the subsequent cuts on a surface. For certain cutting speeds this phase is minimized increasing stability of the system. Stability diagrams show the stable cutting depths as a function of the cutting speed, and thus can be used to determine stable machining conditions without losing productivity. Therefore, in order to fully employ the capacity of a machine tool under stable cutting conditions it is required to obtain the stability lobe diagram for a certain application using a spindle–holder–tool combination.

History of the stability lobe diagrams and chatter vibrations of machine tools extends to the studies of Tobias [1,3] and Tlusty [2,4] which present the basics of regenerative chatter for orthogonal cutting conditions and time invariant process factors, such as the direction of cutting force and chip thickness. Merrit [5] suggested a theory that uses Nyquist stability criterion yielding similar results for the same conditions. However, the stability analysis of milling is complicated due to the rotational tool resulting in continuously changing directional factors and time-varying system dynamics. The approximate analytical

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model for milling stability presented by Tlusty [4] was followed by the time domain simulations [6–8] for prediction of chatter stability in milling. Minis and Yanushevsky [9,10] employed Floquet's theorem and Fourier series for the formulation of milling stability and used Nyquist criterion for the numerical solution. Altintas and Budak [11] presented an analytical milling stability model also by using Floquet's theorem and Fourier series representation of time-varying directional coefficients. They also included the changing dynamics of the machine tool and workpiece in the cutting zone [12]. Regardless of the approach (numerical or analytical), the common point of the models used for generation of stability lobe diagrams is the requirement of the tool point frequency response function (FRF) of the assembly. The well-known relation for the critical width of cut ($b_{\lim}$) for an orthogonal cutting process is given as [13]

$$b_{\lim} = \frac{-1}{2K_{\mathrm{f}}\, m \mathrm{Re}[G(\omega)]}, \tag{1}$$

where K_{f} is the cutting force coefficient in chip thickness direction, m is the average number of teeth in cut, and $\mathrm{Re}[G(\omega)]$ refers to the real part of the resulting FRF at the cutting point. Thus, the transfer function (in the form of point receptance) $G(\omega)$ is required to obtain the stability lobe diagram of a given spindle–holder–tool combination. Using experimental approach, this FRF can be obtained directly by impact testing. A low-mass accelerometer is placed at the tool tip of the assembly and the system is excited at the same point by using an impulse hammer to obtain the tool point FRF through a spectrum analyzer or modal testing software. However, for a different combination of the system components (even when the overhang length of the tool is changed), a new test will be required since the system dynamics will change. Therefore, the use of experimental modal analysis requires considerable time and therefore is not very practical, as it should be repeated for every holder and tool changes.

In order to minimize experimentation, recently researchers attempted to obtain $G(\omega)$ semi-analytically. Schmitz et al. [14–17] implemented the well known receptance coupling theory of structural dynamics in order to couple the dynamics of the spindle–holder assembly and the tool by using the dynamical properties at the holder–tool interface. Thus, it is suggested to make only a single experiment at the holder tip of the machine tool, and the dynamics of the holder is coupled with the analytically obtained tool dynamics, which is modeled as a uniform Euler–Bernoulli beam, in order to obtain the tool point FRF of the complete system. As long as the dynamical properties at holder–tool interface are analytically modeled or experimentally obtained accurately for different clamping conditions, this semi-analytical model can provide accurate results and may save considerable time in applications where only the conditions related to the cutting tool are changed. It has previously been observed that [18,19] tool overhang length itself is a practical parameter to change the dynamics of the system, especially with its dynamic vibration absorber effect which sometimes makes higher overhang lengths more stable than the lower ones. Schmitz et al. also observed the dynamic absorber effect of tool in their studies [14,15]. Duncan et al. [20] focused on the use of this effect in a recent study.

Several improvements have been done on the receptance coupling approach in the following studies. Park et al. [21] included the rotational degree of freedom at the tool holder–tool joint whereas Kivanc and Budak [22] modeled the tool as a two-segment beam considering the changing area moment of inertia for more accurate results. They also studied the effects of the contact length and the clamping torque on the holder–tool contact stiffness and damping properties. Duncan and Schmitz [23] improved the use of receptance coupling approach to handle different holder types using a single experimental measurement.

Medicus and Schmitz [24] worked on the dynamic repeatability of the tool point FRF for holder and tool changes since repeatability is of quite importance in production applications. As the dynamics of the spindle–holder interface also affects the dynamic stiffness at the tool tip, the literature includes studies that point out the importance of this interface, and suggest alternative connection ways [25–28].

In the study presented here, not only the tool, but all components of spindle–holder–tool assembly are modeled analytically and coupled in order to obtain tool point FRF by using receptance coupling and structural modification methods. The details of the mathematical model are given in this paper. The model developed is used for predicting the tool point FRF of an example spindle–holder–tool assembly and the results are compared with those of a finite element software for verification.

2. Mathematical modeling

2.1. Modeling of component segments

In this study, all components of the spindle–holder–tool assembly are modeled as multi-segment Timoshenko beams. Euler–Bernoulli beam model used in the previous studies [14–17,22,23] has been found to be insufficient for modeling the component dynamics at high frequencies because of their low slenderness ratios since this approach neglects the effects of rotary inertia and shear deformation. In this study, Timoshenko beam model [29,30] is used, and the results are compared with those of Euler–Bernoulli formulation, and a considerable improvement has been observed in the predictions for even relatively slender components.

The eigenvalue problem of an m-segment Timoshenko beam yields a characteristic equation expressed in terms of the determinant of a $4m \times 4m$ matrix. As the elements of the matrix are highly nonlinear, even though the problem and the size of the matrix are physically meaningful, the condition number of such a large order matrix generally