International Journal of Fatigue 31 (2009) 776–781

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

A new multiaxial fatigue damage model for various metallic materials under the combination of tension and torsion loadings

Jing Li *, Zhong-ping Zhang, Qiang Sun, Chun-wang Li, Yan-jiang Qiao

The Science Institute, Air Force Engineering University, East Chang Le Road, Xi'an 710051, China

ARTICLE INFO

Article history:
Received 17 December 2007
Received in revised form 2 March 2008
Accepted 4 March 2008
Available online 12 March 2008

Keywords:
Multiaxial fatigue
Critical plane approach
Additional cyclic hardening
Life prediction
Proportional loading and non-proportional loading

ABSTRACT

Based on the critical plane approach, a new damage parameter for multiaxial fatigue damage is presented. Both components of strain and stress are considered in this parameter. Thus, a new multiaxial fatigue damage model is given based on the critical plane approach. The capability of fatigue life prediction for the proposed fatigue damage model is checked against the experimental data of Hot-rolled 45 Steel, S460N Steel, 1045HR Steel, 30CrMnSiNi2A alloy steel, and GH4169 alloy at elevated temperature, and the predicted results are compared with results from common multiaxial fatigue model. It is demonstrated that the proposed criterion gives better satisfactory results for all the five checked materials.

1. Introduction

The members and components of engineering structure in service are usually subjected to non-proportional cyclic loading which lead to changing of the principal stresses and strains directions during a cycle of loading. The additional hardening of material, which is caused by the rotation of the principle stress and strain axes, is considered to have tight relationship to the reduction of fatigue life under non-proportional loading compared with that under proportional loading [1–5]. The key problem in evaluating fatigue damage in these circumstances is the necessity of using multiaxial fatigue damage criteria which are based on aspects of the loading history. In low cycle fatigue, according to the parameters used in the fatigue criteria, the prediction methods can be classified into three categories, namely the equivalent strain approach, energy approach and the critical plane method. Reviews of available multiaxial fatigue life prediction methods are presented by Glinka et al. [6], Mcdiarmid [7], Brown and Miller [8], You and Lee [9], Shang and Wang [10], Shang and Sun et al. [11]. Fatigue analysis using the concept of a critical plane of maximum shear strain is very effective because the critical plane concept is based on the physical observations that cracks initiate and grow on preferred planes.

In the present study, a multiaxial fatigue parameter based on the critical plane concept is proposed, which does not include any material constant. A new multiaxial fatigue model is given to predict multiaxial fatigue life under both proportional loading and non-proportional loading on the basis of this parameter as well.

2. Strain analysis under the combination of tension and torsion loadings

Fig. 1 illustrated a thin-walled tubular specimen subjected to combined tension and torsion loadings. The strain tensor for the thin-walled tubular specimen subjected to axial and torsional fatigue under strain-controlled loading conditions is given by Eq. (1) as

$$\Delta\varepsilon_{ij} = \begin{bmatrix} \Delta\varepsilon_x & 1/2\Delta\gamma_{xy} & 0 \\ 1/2\Delta\gamma_{xy} & -\nu_{\mathrm{eff}}\Delta\varepsilon_x & 0 \\ 0 & 0 & -\nu_{\mathrm{eff}}\Delta\varepsilon_x \end{bmatrix} \tag{1}$$

If the applied strains are sinusoidal, i.e.

$$\Delta\varepsilon_x = \frac{\Delta\varepsilon}{2}\sin\omega t \tag{2}$$

$$\Delta\gamma_{xy} = \frac{\Delta\gamma}{2}\sin(\omega t - \varphi) \tag{3}$$

where φ is the phase angle between the tensional strain and torsion strain. $\Delta\varepsilon$ and $\Delta\gamma$ are the applied tensional and torsion strain range, respectively. In Eq. (1) ν_{eff} is the effective Poisson's ratio which is given by

$$\nu_{\mathrm{eff}} = \frac{\nu_{\mathrm{e}}\Delta\varepsilon_{\mathrm{e}} + \nu_{\mathrm{p}}\Delta\varepsilon_{\mathrm{p}}}{\Delta\varepsilon_{\mathrm{e}} + \Delta\varepsilon_{\mathrm{p}}} \tag{4}$$

* Corresponding author.
E-mail address: lijing02010303@163.com (J. Li).

0142-1123/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijfatigue.2008.03.008

Nomenclature

E	Young's modulus
$\sigma_{0.2}$	yield strength (0.2%)
σ_b	ultimate tensile strength
v_{eff}, v_e	effective and elastic Poisson's ratio, respectively
$\Delta\sigma_n$	normal stress range which is determined by the Ramberg–Osgood relation
b, c	fatigue strength and ductility exponents
σ'_f, ε'_f	fatigue strength and ductility coefficients
K', n'	cyclic strength coefficient and strain hardening exponent
$\Delta\varepsilon_n$	normal strain range on the critical plane
$\Delta\gamma_{max}$	maximum shear strain range
N_f	fatigue life
$\Delta\varepsilon$	applied axial strain range
$\Delta\gamma$	applied shear strain range
λ	strain ratio, $\Delta\gamma/\Delta\varepsilon$
φ	phase angle difference

where v_e and v_p are the elastic and plastic Poisson's ratio, respectively. The axial elastic strain range is calculated using Hooke's law

$$\Delta\varepsilon_e = \frac{\Delta\sigma}{E} \tag{5}$$

The plastic strain range $\Delta\varepsilon_p$ is determined by the Ramberg–Osgood relation

$$\Delta\varepsilon_p = \Delta\varepsilon_x - \frac{\Delta\sigma}{E} \tag{6}$$

In Eqs. (5) and (6), $\Delta\sigma$ is the range of the axial stress. E is modulus of elasticity.

Then the normal strain and the shear strain on the maximum shear plane which make an angle α with the thin-walled tubular specimen axis are given by Kanazawa, Brown and Miller [12]

$$\gamma_{max}(t) = \frac{1}{2}\Delta\varepsilon\sqrt{[\lambda\cos 2\alpha\cos\varphi - (1+v_{eff})\sin 2\alpha]^2 + [\lambda\cos 2\alpha\sin\varphi]^2}\,\sin(\omega t + \eta) \tag{7}$$

$$\varepsilon_n(t) = \frac{1}{4}\Delta\varepsilon\sqrt{[2(1+v_{eff})\cos^2\alpha - 2v_{eff} + \lambda\sin 2\alpha\cos\varphi]^2 + [\lambda\sin 2\alpha\sin\varphi]^2}\,\sin(\omega t - \xi) \tag{8}$$

where

$$\tan\xi = \frac{\lambda\sin 2\alpha\sin\varphi}{(1+v_{eff})\cos 2\alpha + (1-v_{eff}) + \lambda\sin 2\alpha\cos\varphi} \tag{9}$$

$$\tan\eta = \frac{-\lambda\cos 2\alpha\sin\varphi}{\lambda\cos 2\alpha\cos\varphi - (1+v_{eff})\sin 2\alpha} \tag{10}$$

$$\tan 4\alpha = \frac{2\lambda(1+v_{eff})\cos\varphi}{(1+v_{eff})^2 - \lambda^2} \tag{11}$$

$$\lambda = \Delta\gamma/\Delta\varepsilon \tag{12}$$

The phase angle between ε_n and γ_{max} is $(\xi + \eta)$, range between $-\pi/2$ and $\pi/2$. From Eqs. (7) and (8), the $\Delta\gamma_{max}$ and $\Delta\varepsilon_n$ can be obtained as follows:

$$\Delta\gamma_{max} = \Delta\varepsilon\sqrt{[\lambda\cos 2\alpha\cos\varphi - (1+v_{eff})\sin 2\alpha]^2 + [\lambda\cos 2\alpha\sin\varphi]^2} \tag{13}$$

$$\Delta\varepsilon_n = \frac{1}{2}\Delta\varepsilon \times \sqrt{[2(1+v_{eff})\cos^2\alpha - 2v_{eff} + \lambda\sin 2\alpha\cos\varphi]^2 + [\lambda\sin 2\alpha\sin\varphi]^2} \tag{14}$$

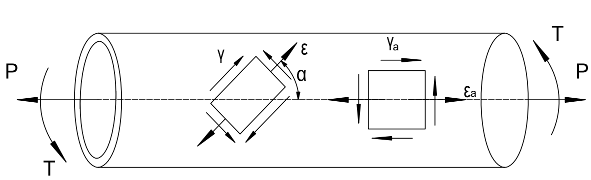

Fig. 1. Strain state of the tension–torsion specimen.

Under the triangle wave loading, Ref. [13] has verified the applicability of the sinusoidal approach for the triangle wave loading. Therefore, it is used to calculate the orientation of the critical plane and the damage parameter in this paper.

3. Multiaxial fatigue damage model

3.1. Proposed parameter and analysis

Critical plane models have been proposed by several researchers. Most of the proposed criteria are given in the form of expressions involving a combination of the stress [7] and strain [14,15] components associated with the critical plane. In general, the strain criterion can be given in the form of the critical plane involving shear and normal strain components, as proposed by Kandile, Brown and Miller (KBM) [15].

$$\frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_n = C \tag{15}$$

where $\Delta\gamma_{max}$ is the maximum shear strain range, $\Delta\varepsilon_n$ is the normal strain range acting on the $\Delta\gamma_{max}$ plane and S is a material constant.

Take 1045HR Steel for example, the relationship between the phase delay and the maximum shear range $\Delta\gamma_{max}$, the normal strain range $\Delta\varepsilon_n$ acting on the maximum shear plane and the normal stress range $\Delta\sigma_n$ which is determined by the Ramberg–Osgood relation are illustrated in Fig. 2a and b, respectively. From Fig. 2a and b, it can be observed that under the same equivalent strain loading, $\Delta\varepsilon_n$ and $\Delta\sigma_n$ acting on the maximum shear plane become larger when the phase delay between axial and torsional loading increases. The experimental fatigue lives presented in Refs [1,11,13,16,17] are usually reduced when the phase angle difference increases. Therefore, it is rational to use the $\Delta\varepsilon_n$ acting on the maximum shear plane and $\Delta\sigma_n$ which is determined by the Ramberg–Osgood relation as the fatigue damage parameters under the multiaxial loading, because they reflect the fact that the fatigue lives are reduced when the phase delay between axial and torsional loading increases.

Based on the critical plane concept, a new multiaxial fatigue damage parameter (LZH), including the stress and strain components acting on the critical plane, is proposed. The maximum shear strain plane is taken as the critical plane.

$$\frac{\Delta\gamma_{max}}{2} + \left(1 + \frac{\Delta\sigma_n}{2\sigma_{0.2}}\right)\Delta\varepsilon_n = f(N_f) \tag{16}$$