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Modified Joint Weibull approach to determine Load Enhancement Factors

James Giancaspro [a,*], Winson Taam [b], Raymond Wong [c]

[a] Department of Civil, Architectural, and Environmental Engineering, University of Miami, 1251 Memorial Drive, Coral Gables, FL 33146-0630, United States
[b] Boeing Phantom Works, P.O. Box 3707, MC 7L-22, Seattle, WA 98124, United States
[c] Boeing Commercial Airplanes, P.O. Box 3707, MC 03-JL, Seattle, WA 98124-2207, United States

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ABSTRACT

Load Enhancement Factor (LEF) and similar statistical methods have been used for decades to reduce the time and cost associated with component-level fatigue testing on aerospace structures. The most common LEF approach was that developed by the Naval Air Development Corporation in the 1980s. Though considered an innovative and novel concept at its conception, this traditional method has a number of limitations that restrict its applicability to only a handful of testing scenarios. The objective of this study was to deal with those restrictions and offer a more comprehensive approach to account for modern advances in statistics, composite materials, and testing technology. The formulation of the new method uses the traditional LEF method as a foundation, but uses a revised set of notation and incorporates a modified Joint Weibull analysis technique to improve its potency. A detailed set of sample calculations using stochastically generated data illustrates how the computations are performed, thus allowing practitioners to reproduce the method using their own data. A short discussion also addresses some common misconceptions regarding the use of Load Enhancement Factors.

1. Introduction

1.1. Background

Composite structures are advantageous over metallic structures in a number of ways, including corrosion resistance, reduced weight, and fatigue performance. However, the modes in which damage initiation and progression (fatigue) occur are complex and extremely difficult to model analytically. As a result, certifying the durability of composite structure relies heavily upon experimental fatigue tests. Since fatigue scatter in composites has been, in general, much higher than in its metallic counterparts, experimental fatigue tests must be conducted on many replicates to achieve the desired levels of reliability. Carrying out such a vast array of experimental tests dramatically increases the cost and time necessary to complete certification. In an attempt to manage the effects of fatigue scatter and to reduce the time and cost associated with testing components containing composite structure, a number of statistically-based fatigue testing approaches have been developed [1–4].

1.2. Traditional approaches and methods

Among the most common approaches, include the Life Factor Approach, Load Factor Approach, and the Load Enhancement Factor Approach (LEF), Fig. 1. The Life Factor approach utilizes test loads similar to those predicted during actual operating conditions of the component, but increases the duration of the test to achieve the desired reliability. Unfortunately, this approach may result in long test durations such as 13 lifetimes, as illustrated by Whitehead et al. [2,3]. In contrast to the Life Factor scheme, the Load Factor approach increases the test loads, but holds the test duration constant. This approach also has a drawback in that achieving the required level of reliability may necessitate increasing the test loads near or even exceeding the static strength of the structural component if the material is susceptible to high levels of fatigue scatter. Furthermore, high fatigue test loads may lead to a change in the fatigue failure mode of the component, a condition that may misrepresent the loading conditions of the actual structure during operation.

To overcome the individual disadvantages of the Load Factor and Life Factor approaches, these factors may be used in concert to achieve both reasonable test durations and test loads that are well below the static strength of the component. Combining these approaches yields a method commonly known as the Load Enhancement Factor. This scheme has been utilized for decades and is commonly regarded as the industry standard for fatigue testing on the component-level for composite structures [5–8]. For instance, Abbott and Kolarik [7], Harris et al. [9], and Lameris [10] followed the LEF values developed by this approach literally, and the method has been incorporated into MIL-HDBK-17-3F [11].

The authors of the original Load Enhancement Factor (LEF) method sought to develop a statistically based approach with

* Corresponding author. Tel.: +1 305 284 1006; fax: +1 305 284 3492.
E-mail address: jwgiancaspro@miami.edu (J. Giancaspro).

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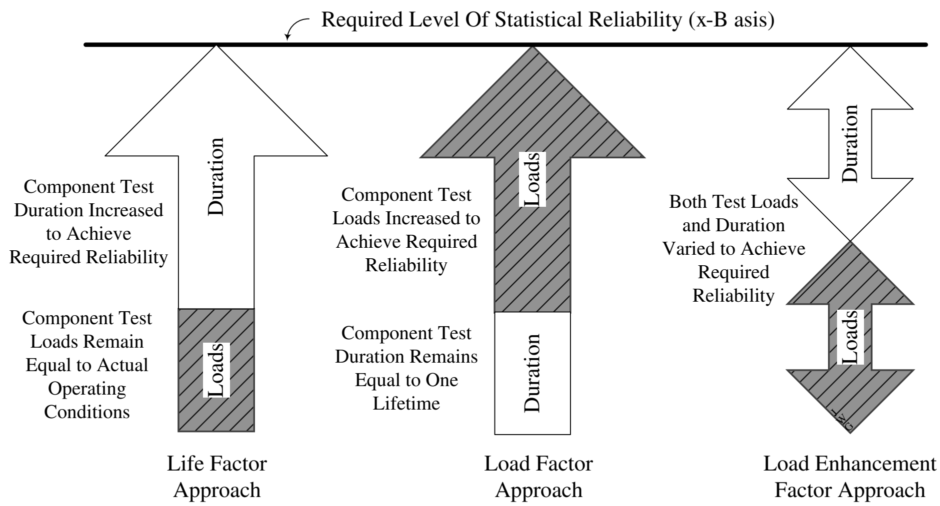

Fig. 1. Life, load, and Load Enhancement Factor approaches.

one purpose in mind: "... to increase the applied loads in the fatigue certification tests so that the same level of reliability can be achieved with a shorter test duration [3]."

By using a Load Enhancement Factor, more economical fatigue tests can be conducted while still maintaining the desired levels of statistical reliability and confidence. Besides the unconventional use of notation in the derivation, the original LEF method in [2] also possesses a number of hidden limitations. First, consider their Joint Weibull analysis Eq. (10) in [2] to estimate the shape parameter,

$$\sum_{i=1}^{M}\left(\frac{\sum_{j=1}^{n_i} x_{ij}^{\hat{a}} \ln x_{ij}}{\sum_{j=1}^{n_i} x_{ij}^{\hat{a}}}\right) - \frac{M}{\hat{a}} - \left[\sum_{i=1}^{M} \frac{\sum_{j=1}^{n_{fi}} \ln x_{ij}}{n_{fi}}\right] = 0 \qquad (1)$$

where M is the total number of groups (stress levels), x_{ij} is the jth data point in the ith group of data, $n_i(i = 1, 2, \ldots, M)$ is the number of data points in the ith group of data, $n_{fi}(i = 1, 2, \ldots, M)$ is the number of failures in the ith group of data, and $\hat{a}$ is the shape parameter. By rewriting this equation, one would obtain the following,

$$\frac{\hat{a}\left[\sum_{i=1}^{M}\left(\frac{\sum_{j=1}^{n_i} x_{ij}^{\hat{a}} \ln x_{ij}}{\sum_{j=1}^{n_i} x_{ij}^{\hat{a}}}\right) - \left[\sum_{i=1}^{M} \frac{\sum_{j=1}^{n_{fi}} \ln x_{ij}}{n_{fi}}\right]\right]}{M} = 1 \qquad (2)$$

In this form, the denominator on the left side of the equation is M, the total number of groups. Therefore, the left side of the equation is some form of an average value of M groups. The equation is only valid for stress levels with the same number of tested and failed coupons. In addition, this equation implies that n_{f_i} must be equal for all M stress levels. Scrutiny of Eq. (2) would reveal the following limitations:

- Total number of coupon replicates in each stress level must be equal.
- Number of coupons that reach the failure condition must be equal in each stress level.
- Number of coupons that reach the run-out condition must be equal in each level.
- LEF cannot be computed if there are insufficient residual strength data.

These implicit assumptions severely restrict the use of the LEF method. For example, consider the stochastically generated data in Table 1, which represents coupon-level fatigue test results in which 26 coupons were tested using five distinct fatigue stress levels. Here, n denotes number of coupons while i designates the group number. To employ the traditional LEF method outlined in Whitehead et al. [3], several requirements must be first verified. First, at least two coupons in the testing program must reach the run-out condition and be tested for residual strength. Since groups 3, 4, and 5 contain a total of 11 specimens tested for residual strength, this condition is easily met. Secondly, all stress levels must contain equal number of total coupons, $n_i = n_{i+1} = n_{i+2} = \cdots$, which is clearly not true; only groups 1 and 2 match this criterion $n_1 = n_2$. The next two requirements stipulate that all groups must contain the same number of coupons reaching failure, namely, $n_{fi} = n_{f(i+1)} = n_{f(i+2)} = \cdots$, and the same number of coupons reaching the run-out condition, $n_{ri} = n_{r(i+1)} = n_{r(i+2)} = \cdots$. Neither of these conditions is satisfied. Since not all four requirements are satisfied, the traditional LEF method may *not* be employed to calculate the required Load Enhancement Factors.

Table 1
Example fatigue data for coupon-level testing

Group number, i	Specimen number	Cyclic stress, S (MPa)	Total number of cycles	Residual strength (ksi)	Number of coupons		
					Total, n_i	Run-out, n_{ri}	Failed, n_{fi}
1	1	57.96	1,000,000	90.85	5	5	0
	2	57.96	1,000,000	92.60			
	3	57.96	1,000,000	92.60			
	4	57.96	1,000,000	64.50			
	5	57.96	1,000,000	95.30			
2	6	62.79	1,000,000	83.99	5	5	0
	7	62.79	1,000,000	85.88			
	8	62.79	1,000,000	86.79			
	9	62.79	1,000,000	89.18			
	10	62.79	1,000,000	89.69			
3	11	65.69	493,004	–	7	5	2
	12	65.69	976,504	–			
	13	65.69	1,000,000	77.71			
	14	65.69	1,000,000	77.75			
	15	65.69	1,000,000	83.20			
	16	65.69	1,000,000	85.00			
	17	65.69	1,000,000	85.33			
4	18	67.62	128,657	–	5	0	5
	19	67.62	398,586	–			
	20	67.62	550,397	–			
	21	67.62	656,841	–			
	22	67.62	799,445	–			
5	23	72.45	116,720	–	4	0	4
	24	72.45	142,619	–			
	25	72.45	223,825	–			
	26	72.45	288,776	–			