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Short Communication

On the accurate calculation of milling stability limits using third-order full-discretization method

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ABSTRACT

Based on third-order Newton's Interpolation theory, this paper proposed one method to compute milling stability. The machining is first considered as a dynamic process expressed by a mathematical equation, and this equation integrates the regenerative effect utilizing a time delay item. The time period is discretized as a series of small elements. Then, in each time element, the third-order Newton's interpolation algorithm is used to approximate the state item of the equation. The time-period and time-delay items are expressed by liner-interpolation. After equation items are expressed using the interpolation method on the time period, a matrix denoting the machining system is built. Taking advantage of the matrix, the stability of milling process is investigated, and the convergence feature of the proposed method is also analyzed. Finally, examples of 1-dof and 2-dof dynamic systems are conducted and the comparison results show that the method is effective.

1. Introduction

In milling process, machining chatter has negative influence on the machined surface quality. How to avoid vibration is a key issue to improve machining efficiency and accuracy. The dynamic process described by delay-differential equations (DDEs) [1,2] embraces regeneration of instantaneous uncut chip thickness, and the stability prediction based on DDEs can be used to find the relations between axial cutting depth, radial cutting depth and the rotation speed of the machine via stability lobe diagrams.

Except the experimental method [3] and the experimental–analytical method [4], numerical algorithms of predicting stability lobes have been developed. Altintas and Budak [3–8] have made great effort on this aspect. Their basic way to predict stability lobes is translating DDEs from time domain to frequency domain using Laplace Transform. And then the limit axial cutting depth and corresponding rational speed are calculated utilizing real and image part of the characteristic equation of the system in frequency domain under the premise of giving radial cutting depth. Utilizing the method, Kivanc and Budak [9,10] took finite element analysis(FEA) as a tool to carry out the static and dynamic analysis of tools with different geometry and material, then predict stablilty lobes. Using the same idea as Kivanc, Ozlu and Budak [11,12] proposed a method for predicting stablilty limits in turning and boring operations.

In addition, the numerical algorithms in time domain are also developed. Insperger and Stépán [1] proposed a significant updated semi discretization method to predict the stability lobes. And the result has been proved efficient by Catania and Mancinelli [13]. In their milling machine-tool model, the system is divided into two parts. The first part contains the machine frame and the spindle, the other is the cutter. The advantage of this method is it does not need experiment tests when changing cutter. Smith and Tlusty [14], Zhao and Balachandran [15] also developed numerical methods to predict the stablilty lobes. Recently, Ding et al. [16] introduced a numerical integration scheme to obtain the stablity lobes. Then they [17,18] first developed one-order and second-order full-discretization methods which have shown good advantages to predict stablilty lobes. Tamas [19] also made some comparations between the semi-discretization method (SDM) and the full-discretization method (FDM).

The purpose of this short communication is to update the FDM method by a third-order Newton's interpolation theory and make detailed comparisons with the existing FDM method and the SDM method to show the characteristics and necessity of developing a three-order FDM method.

2. Mathmatical modal of third-order FDM

The dynamic system of the machine-tool with regenerative effect can be expressed by a n-dimensional equation in the state-space as [17]

$$\dot{\mathbf{x}}(t) = \mathbf{A}_0\mathbf{x}(t) + \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{x}(t-T) \quad (1)$$

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where $\mathbf{A}_0$ is the constant matrix standing for the time invariants of the dynamic system. $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are two periodic functions with the period T. $\mathbf{x}(t)$ is the state variable of the system. The solution of this time delay differential equation is as follows:

$$\mathbf{x}(t)=e^{\mathbf{A}_0(t-t_0)}\mathbf{x}(t_0)+\int_{t_0}^{t}e^{\mathbf{A}_0(t-\tau)}[\mathbf{A}(\tau)\mathbf{x}(\tau)+\mathbf{B}(\tau)\mathbf{x}(\tau-T)]d\tau \tag{2}$$

In order to obtain the stability lobes of the dynamic system, the first step is to divide the period T into m time-intervals, and for each time interval the solution of the time delay differential equation can also use the form of Eq. (2). Thus, for the ith time interval, the solution is expressed as

$$\mathbf{x}(t)=e^{\mathbf{A}_0(t-ih)}\mathbf{x}(t_0)+\int_{ih}^{t}e^{\mathbf{A}_0(t-\tau)}[\mathbf{A}(\tau)\mathbf{x}(\tau)+\mathbf{B}(\tau)\mathbf{x}(\tau-T)]d\tau \tag{3}$$

with $h=T/m$ and $t\in[ih,(i+1)h]$. Let $\mathbf{x}_i$ stand for $\mathbf{x}(ih)$. Then $\mathbf{x}_{(k+1)}$ can be given as

$$\mathbf{x}_{(k+1)}=e^{\mathbf{A}_0h}\mathbf{x}_k+\int_0^h e^{\mathbf{A}_0(t-\tau)}[\mathbf{A}(\varepsilon)\mathbf{x}(\varepsilon)+\mathbf{B}(\varepsilon)\mathbf{x}(\varepsilon-T)]d\varepsilon \tag{4}$$

where $\varepsilon=\tau-kh$, $\varepsilon\in[0,h]$.

The next step is to substitute Eq. (4) using one- and third-order interpolation theories. For kth time interval, $\mathbf{A}(\varepsilon)$, $\mathbf{B}(\varepsilon)$ and $\mathbf{x}(\varepsilon-T)$ are interpolated by two boundaries of the kth time interval $[\mathbf{A}_k, \mathbf{A}_{(k+1)}]$, $[\mathbf{B}_k, \mathbf{B}_{(k+1)}]$ and $[\mathbf{x}_{(k-m)}, \mathbf{x}_{(k+1-m)}]$ correspondingly as

$$\mathbf{A}(\varepsilon)\approx\frac{\varepsilon}{h}\mathbf{A}_{(k+1)}+\frac{h-\varepsilon}{h}\mathbf{A}_k \tag{5}$$

$$\mathbf{B}(\varepsilon)\approx\frac{\varepsilon}{h}\mathbf{B}_{(k+1)}+\frac{h-\varepsilon}{h}\mathbf{B}_k \tag{6}$$

$$\mathbf{x}(\varepsilon-T)\approx\frac{\varepsilon}{h}\mathbf{x}_{(k+1-m)}+\frac{h-\varepsilon}{h}\mathbf{x}_{(k-m)} \tag{7}$$

where $\mathbf{x}(\varepsilon)$ can be approximated by third-order Newton's interpolation equation using $\mathbf{x}_{(k+1)}$, $\mathbf{x}_k$, $\mathbf{x}_{(k-1)}$ and $\mathbf{x}_{(k-2)}$ as follows:

$$\begin{aligned}
&\mathbf{x}(\varepsilon)\approx a\mathbf{x}_{(k+1)}+b\mathbf{x}_k+c\mathbf{x}_{(k-1)}+d\mathbf{x}_{(k-2)}\\
&a\approx\frac{\varepsilon^3+3\varepsilon^2h+\varepsilon h^2}{6h^3}\\
&b\approx\frac{\varepsilon^2+3\varepsilon h+h^2}{2h^2}-\frac{\varepsilon^3+3\varepsilon^2h+\varepsilon h^2}{2h^3}\\
&c\approx\frac{\varepsilon+2h}{h}-\frac{\varepsilon^2+3\varepsilon h+h^2}{h^2}+\frac{\varepsilon^3+3\varepsilon^2h+\varepsilon h^2}{2h^3}\\
&d\approx\frac{h-\varepsilon+2h}{h}+\frac{\varepsilon^2+3\varepsilon h+h^2}{2h^2}-\frac{\varepsilon^3+3\varepsilon^2h+\varepsilon h^2}{6h^3}
\end{aligned} \tag{8}$$

Then $\mathbf{A}(\varepsilon)$, $\mathbf{B}(\varepsilon)$, $\mathbf{x}(\varepsilon-T)$ and $\mathbf{x}(\varepsilon)$ in Eq. (4) are substituted by Eq. (5–8), and after simplifying, Eq. (4) can be expressed using

$$\begin{aligned}\mathbf{x}_{(k+1)}=&(\mathbf{I}-\mathbf{F}_1)^{-1}(\mathbf{F}_0+\mathbf{F}_2)\mathbf{x}_k+(\mathbf{I}-\mathbf{F}_1)^{-1}\mathbf{F}_3\mathbf{x}_{(k-1)}+(\mathbf{I}-\mathbf{F}_1)^{-1}\mathbf{F}_4\mathbf{x}_{(k-2)}\\
&+(\mathbf{I}-\mathbf{F}_1)^{-1}\mathbf{F}_{m-1}\mathbf{x}_{(k+1-m)}+(\mathbf{I}-\mathbf{F}_1)^{-1}\mathbf{F}_m\mathbf{x}_{(k-m)}\end{aligned} \tag{9}$$

$$\mathbf{F}_0=\mathbf{f}_0=e^{\mathbf{A}_0h} \tag{10}$$

$$\begin{aligned}\mathbf{F}_1=&\mathbf{f}_0(\mathbf{f}_5+3h\mathbf{f}_4+h^2\mathbf{f}_3)\mathbf{A}_{(k+1)}/6h^4\\&+\mathbf{f}_0(2h^3\mathbf{f}_2+h^2\mathbf{f}_3-2h\mathbf{f}_4-\mathbf{f}_5)\mathbf{A}_k/6h^4\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{F}_2=&\mathbf{f}_0(\mathbf{f}_4+3h\mathbf{f}_3+2h^2\mathbf{f}_2)\mathbf{A}_{(k+1)}/2h^3\\&+\mathbf{f}_0(6h^3\mathbf{f}_1+3h^2\mathbf{f}_2-6h\mathbf{f}_3-\mathbf{f}_4)\mathbf{A}_k/6h^3-3\mathbf{F}_1\end{aligned} \tag{12}$$

$$\begin{aligned}\mathbf{F}_3=&\mathbf{f}_0(\mathbf{f}_3+2h\mathbf{f}_2)\mathbf{A}_{(k+1)}/h^2\\&+\mathbf{f}_0(2h^2\mathbf{f}_1-h\mathbf{f}_2-\mathbf{f}_3)\mathbf{A}_k/h^2-2\mathbf{F}_2-3\mathbf{F}_1\end{aligned} \tag{13}$$

$$\mathbf{F}_4=\mathbf{f}_0\frac{\mathbf{f}_2}{h}\mathbf{A}_{(k+1)}+\mathbf{f}_0\left(\mathbf{f}_1-\frac{\mathbf{f}_2}{h}\right)\mathbf{A}_k-\mathbf{F}_2-\mathbf{F}_1-\mathbf{F}_3 \tag{14}$$

$$\mathbf{F}_{m-1}=\mathbf{f}_0\mathbf{f}_3\mathbf{B}_{(k+1)}/h^2+\mathbf{f}_0(h\mathbf{f}_2-\mathbf{f}_3)\mathbf{B}_k/h^2 \tag{15}$$

$$\mathbf{F}_m=\frac{\mathbf{f}_0(h\mathbf{f}_2-\mathbf{f}_3)\mathbf{B}_{(k+1)}}{h^2}+\frac{\mathbf{f}_0(h^2\mathbf{f}_1-2h\mathbf{f}_2+\mathbf{f}_3)\mathbf{B}_k}{h^2} \tag{16}$$

where $\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{f}_3$, $\mathbf{f}_4$ and $\mathbf{f}_5$ in Eqs. (11–16) can be expressed using the following forms:

$$\mathbf{f}_1=\int_0^h e^{-\mathbf{A}_0\varepsilon}d\varepsilon \tag{17}$$

$$\mathbf{f}_2=\int_0^h e^{-\mathbf{A}_0\varepsilon}\varepsilon d\varepsilon \tag{18}$$

$$\mathbf{f}_3=\int_0^h e^{-\mathbf{A}_0\varepsilon}\varepsilon^2 d\varepsilon \tag{19}$$

$$\mathbf{f}_4=\int_0^h e^{-\mathbf{A}_0\varepsilon}\varepsilon^3 d\varepsilon \tag{20}$$

$$\mathbf{f}_5=\int_0^h e^{-\mathbf{A}_0\varepsilon}\varepsilon^4 d\varepsilon \tag{21}$$

All of forms can be evaluated by $\mathbf{f}_0$ in Eq. (10) using the following equations:

$$\mathbf{f}_1=\mathbf{A}_0^{-1}(\boldsymbol{I}-\mathbf{f}_0^{-1}) \tag{22}$$

$$\mathbf{f}_2=\mathbf{A}_0^{-1}(\mathbf{f}_1-h\mathbf{f}_0^{-1}) \tag{23}$$

$$\mathbf{f}_3=\mathbf{A}_0^{-1}(2\mathbf{f}_2-h^2\mathbf{f}_0^{-1}) \tag{24}$$

$$\mathbf{f}_4=\mathbf{A}_0^{-1}(3\mathbf{f}_3-h^3\mathbf{f}_0^{-1}) \tag{25}$$

$$\mathbf{f}_5=\mathbf{A}_0^{-1}(4\mathbf{f}_4-h^4\mathbf{f}_0^{-1}) \tag{26}$$

Then Eq. (9) is expressed using matrix form

$$\mathbf{Y}_{k+1}=\mathbf{D}_k\mathbf{Y}_k \tag{27}$$

$$\begin{bmatrix}
(\mathbf{I}-\mathbf{F}_1)^{-1}(\mathbf{F}_0+\mathbf{F}_2) & (\mathbf{I}-\mathbf{F}_1)^{-1}\mathbf{F}_3 & (\mathbf{I}-\mathbf{F}_1)^{-1}\mathbf{F}_4 & \cdots & \mathbf{0} & (\mathbf{I}-\mathbf{F}_1)^{-1}\mathbf{F}_{m-1} & (\mathbf{I}-\mathbf{F}_1)^{-1}\mathbf{F}_m\\
\mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I} & \mathbf{0}
\end{bmatrix}\mathbf{D}_K$$

$$\mathbf{Y}_{k+1}=\begin{bmatrix}\mathbf{x}_{k+1} & \mathbf{x}_k & \mathbf{x}_{k-1} & \cdots & \mathbf{x}_{k-m+1}\end{bmatrix}^T$$

During one period, the $\mathbf{Y}_m$ at time $t=T$ can be expressed by $\mathbf{Y}_0$ at time $t=0$ using

$$\mathbf{Y}_m=\mathbf{D}_{m-1}\mathbf{D}_{m-2}\mathbf{D}_{m-3}\cdots\mathbf{D}_0\mathbf{Y}_0 \tag{28}$$

Finally stability lobes can be predicted using transformation matrix during one period based on Floquet theory. The transformation matrix during one period $\mathbf{D}$ is

$$\mathbf{D}=\mathbf{D}_{m-1}\mathbf{D}_{m-2}\mathbf{D}_{m-3}\cdots\mathbf{D}_0 \tag{29}$$

If $(\mathbf{I}-\mathbf{F}_1)$ has no inverse matrix, then the Moore–Penrose generalized inverse matrix is used as substitution [17].