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A design procedure for assessing low cycle fatigue life under proportional and non-proportional loading

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Abstract

This paper discusses the design assessment for structural components subjected to proportional and non-proportional loading. Multiaxial low cycle fatigue lives are influenced by stress and strain multiaxiality, their non-proportionality and a material property that relates to the degree of additional hardening. Many low cycle fatigue studies under proportional and non-proportional loading were carried out in laboratories, but a little study discussed the application of the results obtained in laboratories to an actual design for structural components. This paper proposes a fatigue life assessment for structural components subjected to proportional and non-proportional low cycle fatigue loading. The assessment provides a simple method for evaluating principal strain range, strain multiaxiality and strain non-proportionality. This paper also discusses low cycle fatigue parameters suitable for the life assessment of structural components subjected to multiaxial loading.

Keywords: Low cycle fatigue; Multiaxial loading; Non-proportional loading; Life prediction; Design criteria

1. Introduction

Developing an appropriate design criterion for multiaxial low cycle fatigue (LCF) is one of the key issues for guaranteeing the reliability of components and structures undergoing multiaxial LCF damage. Many LCF stress and strain parameters [1–3] were proposed for correlating LCF lives under tension-torsion proportional loading using hollow or solid cylinder specimens. However, this testing method only enabled LCF tests with a limited range of strain multiaxiality and experimental studies in much wider range of strain multiaxiality were needed. The authors carried out the multiaxial LCF tests in a full range of strain multiaxiality using cruciform specimens [4,5] and demonstrated that suitable strain parameters for correlating proportional LCF lives in the range were the maximum principal strain and the equivalent strain based on crack opening displacement (COD strain). COD strain was derived from a center cracked plate specimen subjected to biaxial loading but it was successfully applied to the LCF life prediction for plain specimens because crack propagation period shares a major part of failure life in LCF.

Critical issues are still continuing on the parameter for assessing LCF fatigue lives under non-proportional loading [2,6–12]. Parameters proposed for correlating LCF lives under non-proportional loading so far are classified into three categories. They are energy parameters by multiplying stress and strain, critical plane parameters and strain parameters taking account of strain non-proportionality. The Smith–Watson–Topper parameter [6] is a representative energy parameter which is expressed a product of stress and strain. The Socie–Fatami parameter [8] has been frequently discussed as a critical plane parameter that considered stress and strain applied on a critical shear plane. The Wang–Brown parameter [9] and the non-proportional strain [10,11] are a parameter to assess LCF lives under non-proportional loading written with only strain. Many parameters have been proposed to assess LCF lives under non-proportional loading, but no definite conclusion appeared to be reached because of no sufficient experimental data for the demonstration. Recently, the authors generated systematic LCF data under non-proportional loading and showed that the energy parameter and the non-proportional strain were a suitable parameter for assessing LCF fatigue lives under various non-proportional strain histories [10,11]. However, these parameters still remain in a laboratory level

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Nomenclature

$\varepsilon_1(t)$, $\varepsilon_2(t)$, $\varepsilon_3(t)$	maximum, medium and minimum principal strains at time t, respectively
$\varepsilon I(t)$	maximum absolute value of principal strain given by $\varepsilon I(t)=\text{Max}[\lvert\varepsilon_1(t)\rvert, \lvert\varepsilon_3(t)\rvert]$
εI_{max}	maximum value of $\varepsilon I(t)$ in a cycle
ε_{1max}, ε_{3max}	maximum and minimum principal strains at the time of $\varepsilon I(t)=\varepsilon I_{max}$
$\xi(t)$	angle between εI_{max} and $\varepsilon I(t)$
$\Delta\varepsilon I$	maximum principal strain range
$\Delta\varepsilon_{ASME}$	equivalent strain range defined in code case, ASME Section III, Division 1 NH
$\Delta\varepsilon^*$	equivalent strain range based on COD
$\Delta\varepsilon_{NP}$	non-proportional strain range
ϕ	principal strain ratio ($=\varepsilon_3/\varepsilon_1$ or $\varepsilon_1/\varepsilon_3$)
f_{NP}, f_{NP}^*	non-proportional factor
N_f	number of cycles to failure
N_f^U	number of cycles to failure in uniaxial test
N_f^M	number of cycles to failure in multiaxial proportional test
N_f^N	number of cycles to failure in non-proportional test
C_a^M	critical value for judging whether loading is uniaxial or multiaxial
C_a^N	critical value for judging whether loading is multiaxial or non-proportional

and LCF life assessment applicable to practical components and structures is still an open question.

This paper proposes the definitions of principal strain range and strain non-proportionality, and briefly summarizes the strain parameters for correlating LCF lives under proportional and non-proportional loading. This paper also discusses the design flow for components and structures subjected to proportional and non-proportional loading based on the multiaxial strain parameters previously proposed.

2. Definition of strain multiaxiality

This paper employs the principal strain ratio, ϕ, defined below to express the strain multiaxiality for proportional loading. This paper confines the discussion to a plane stress state because cracks mostly initiated from free surface in LCF tests.

$$\phi = \begin{cases} \varepsilon_1/\varepsilon_3 \text{ for } \lvert\varepsilon_1\rvert \le \lvert\varepsilon_3\rvert \\ \varepsilon_3/\varepsilon_1 \text{ for } \lvert\varepsilon_1\rvert > \lvert\varepsilon_3\rvert \end{cases} \tag{1}$$

ε_1 and ε_3 are the maximum and minimum principal strains, respectively. In proportional LCF tests, ϕ has a constant value in a cycle.

The principal strain ($\varepsilon I(t)$) is also defined as the maximum absolute value of the maximum or minimum principal strain as,

$$\varepsilon I(t) = \text{Max}[\lvert\varepsilon_1(t)\rvert,\ \lvert\varepsilon_3(t)\rvert] \tag{2}$$

In the equation, $\varepsilon_1(t)$ and $\varepsilon_3(t)$ are the maximum and minimum principal strains at time t, respectively. The "Max" denotes taking a lager strain from the two strains in the bracket. The maximum value of $\varepsilon I(t)$ is taken as the maximum principal strain (εI_{max}) as follows,

$$\varepsilon I_{max} = \varepsilon I(t_0) = \text{Max}[\varepsilon I(t)] \tag{3}$$

where t_0 is the time giving εI_{max}.

In order to describe the rotation of principal strain direction under non-proportional loading, the angle ($\xi(t)/2$) is introduced. The rotation angle is defined as the angle between the εI_{max} and $\varepsilon I(t)$ directions as illustrated in Fig. 1(a). The directions of εI_{max} and $\varepsilon I(t)$ are taken as the directions of ε_{1max} and $\varepsilon_1(t)$ when $\varepsilon I_{max}=\lvert\varepsilon_{1max}\rvert$ or as the directions of ε_{3max} and $\varepsilon_3(t)$ when $\varepsilon I_{max}=\lvert\varepsilon_{3max}\rvert$. ε_{1max} and ε_{3max} are the maximum and minimum principal strains at the time giving εI_{max} during a cycle.

Fig. 1(b) is a polar figure of εI(t) in a cycle, where the radius is the amplitudes of $\varepsilon I(t)$, and $\xi(t)$ shows the angle between εI_{max} and $\varepsilon I(t)$ directions. Note that the rotation angle in the polar figure, Fig. 1(b), has a double magnitude compared to that in the specimen shown in Fig. 1(a). In the polar figure shown in Fig. 1(b), we can find that the maximum principal strain, εI_{max}, has the maximum radius, and the time, t_0, is the time giving the εI_{max}. If the $\xi(t)/2$ takes either 0 or $\pi/2$, the loading is proportional and the other cases are non-proportional loading.

The maximum principal strain range ($\Delta\varepsilon I$) is defined as the maximum span of the polar figure, Fig. 1(b), and is equated as,

$$\Delta\varepsilon I = \text{Max}[\varepsilon I_{max} - \cos\xi(t)\varepsilon I(t)] \tag{4}$$

Fig. 2 shows the circular strain path, a sinusoidal strain wave with a phase shift of 90° between tension and torsion, where the axial strain range ($\Delta\varepsilon$) is equivalent to the shear strain range

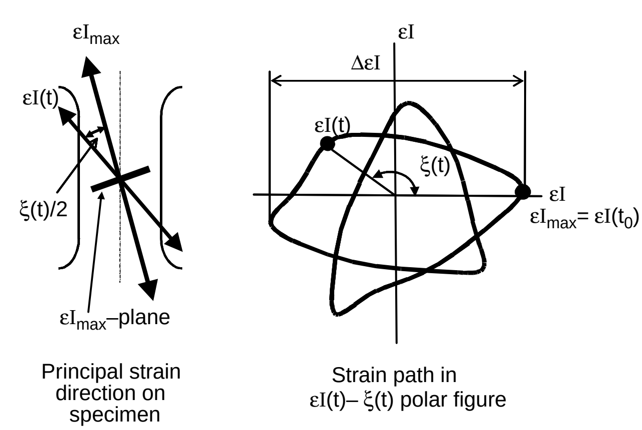

Fig. 1. Schematic of $\varepsilon I(t)$, $\xi(t)$ and $\Delta\varepsilon I$. (a) Principal direction. (b) Definition of strain range.