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Buckling analysis of piezoelectric cylindrical composite panels reinforced with carbon nanotubes

Mohsen Nasihatgozar [a,*], Vahid Daghigh [b,**], Milad Eskandari [a], Kamran Nikbin [c], Andy Simoneau [d]

[a] Department of Mechanical Engineering, Kashan Branch, Islamic Azad University, Kashan, Iran
[b] Young Researchers and Elite Club, Jasb Branch, Islamic Azad University, Delijan, Iran
[c] Department of Mechanical Engineering, Imperial College, London, UK
[d] Department of Mechanical Engineering, University of New Brunswick, Fredericton, Canada

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ABSTRACT

The current article analyses the buckling response of piezoelectric cylindrical composite panels reinforced with carbon nano-tubes subjected to axial load. Classical laminated plate theory (CLPT) is employed to reach stress and displacement correlations embracing mechanical and magnetic terms. Stress–strain equations for piezoelectric cylindrical panels reinforced with carbon nanotubes are then written by using Mori–Tanaka method. The coupled electro-mechanical governing equations, Donnell theory as well as minimum potential energy method are thereafter utilized to calculate buckling loads and modes. The effects of such parameters as volume fraction of nano-tube, geometrical characteristics as well as two loading types of axial and biaxial on buckling load of the composite panel are investigated. The results show that increasing the volume fraction of nano-tube eventuates in increasing the buckling load. The results were compared with those already available in the existed literature using different shell theories for isotropic and composite cylindrical panels and a very good agreement was observed.

1. Introduction

Composite materials are known as new materials enjoying superior properties as compared to traditional materials [1-2]. One of the constituents which can be used to reinforce composites is carbon nanotube (CNT).

Carbon nano-tubes (CNT) were firstly discovered by Iijima [3]. It stimulated many researchers to pay attention to the mechanical properties of CNT as well as the composite materials in which CNT are used as reinforcements ever since. In addition, advanced technologies in a wide variety of industries use piezoelectric materials as sensors or actuators. Thus, taking full advantage of the capabilities of the materials contained such constituents requires an informed understanding of their mechanical response [4].

Arani et al. [5] investigated the buckling analysis of single-walled carbon nanotubes (SWCNT) reinforced rectangular laminated composite plates. The classical laminate theory and the third order shear deformation theory for moderately thick laminate plates were employed to develop the model. The Mori–Tanaka method was also used for the nano-tubes calculations. The critical buckling loads were obtained for different lay-up and boundary conditions using the finite element and the analytical methods and good agreement was observed. It was found that the orientation angle of 45 resulted in the highest critical buckling load. Mosallaie Barzoki et al. [6] considered doublewalled boron nitride nanotubes reinforced piezoelectric polymeric cylindrical shell with an elastic core subjected to electro-thermo-mechanical torsional buckling. The strains based on Donnell theory was used to calculate the critical buckling load. They concluded that buckling strength increased by harder foam cores. Ranjbartoreh et al. [7] analyzed buckling behavior of double-walled carbon nanotubes (DWCNTs) with surrounding elastic medium subjected to axial pressure. Classical theory of plates and shells as well as the Galerkin method are employed for the analysis and numerical solution method. The results showed that double-walled carbon nano-tube enjoyed higher critical axial force as compared to single-walled carbon nanotube however the critical axial pressure of DWCNT is less than that of SWCNT. Arani et al. [8] presented the thermal effect on buckling response of DWCNT which is embedded on the Pasternak foundation subjected to a uniform external pressure. They investigated some effects such as small scale effect, the effect of temperature change in high-temperature environment and van der Waals forces between inner and outer tubes all based on the nonlocal continuum cylindrical shell theory. They found that the axial half wave number directly

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[*] Corresponding author.
[**] Corresponding author. Tel.: +98 8644263850-58; fax: +98 8644263512.
E-mail addresses: mohsen.nasihat@gmail.com (M. Nasihatgozar), Vahid.daghigh_del@yahoo.com (V. Daghigh).

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played role in the nonlocal critical buckling pressure for any specific circumferential wave number. In addition, the influence of temperature change on the critical buckling pressure was found to be trivial for stiff elastic medium. Jabbari et al. [9] analyzed the buckling behavior of porous circular plate bounded with the layers of piezoelectric actuators subjected to uniform radial compressive loading with the clamped edge condition. Variational formulation was used to derive general mechanical equilibrium and stability equations yielding the equations of the piezoelectric porous plate. The influences of piezoelectric layers on some parameters such as the buckling load of plate, variation of porosity and piezoelectric layer-to-porous plate thickness ratio were investigated. In a subsequent research work, Jabbari et al. [10] studied the thermal buckling of radially porous circular plate with piezoelectric actuator layers using first order shear deformation theory. The circular plate subjected to temperature was considered and the pertinent closed form solution was achieved. The equations of the piezoelectric porous plate were used similar to the previous research work. It was found that the critical temperature ascended both by increasing piezoelectric thickness and by increasing porous plate thickness. Khorshidvand et al. [11] took into account the buckling behavior of porous circular plate bounded with piezoelectric sensor–actuator patches subjected to uniform radial compressive loading with the clamped edge condition. Variational formulation was used to derive general mechanical equilibrium and stability equations yielding the equations of the piezoelectric porous plate. Some effects such as variation of porosity, piezoelectric layer-to-porous plate thickness ratio as well as the influence of piezoelectric layers on the buckling load of the plate were studied. The porous plate with the sensor–actuator patches was found to be more stable than with the porous plate with two actuators. Abdollahi et al. [12] used higher-order shear and normal deformable plate theory to analyze the buckling behavior of functionally graded piezoelectric rectangular plates. Two closed and open–closed circuits were considered as electrical conditions. Nonlinear governing equations for buckling of thick functionally graded rectangular plates were achieved using the principle of minimum total potential energy and the variational approach. Finally the critical buckling load was obtained by solving analytically the Maxwell and stability equations for a simply supported thick plate which leaded to obtain the critical buckling load. The influenceability of critical buckling load for some parameters such as loading conditions, aspect ratio, thickness and material properties were investigated. Mosallaie Barzoki et al. [13] considered a composite cylindrical shell made of polyvinylidene fluoride (PVDF) for its nonlinear buckling response using a two-dimensional smart model subjected to combined electro-thermo-mechanical loading. Donnell theory and the first order shear deformation theory were used to obtain the nonlinear strain terms. To attain the critical buckling load was calculated using harmonic differential quadrature method (HDQM) for clamped supported mechanical as well as free electric potential boundary conditions at both ends of the smart cylinder. Results showed that considering the piezoelectric effect culminated in the critical buckling load increase. Chen et al. [14] carried out a research work on the buckling and stability of piezoelectric viscoelastic nanobeam subjected to van der Waals. Galerkin method and Euler–Bernoulli hypothesis were used to establish the static and dynamic governing equations.

The effect of some parameters such as electrostatic load, van der Waals force, inner damping on the post-buckling as well as the principal region of instability were studied. Zhang et al. [15] investigated the buckling response of functionally graded carbon nano-tube (FG-CNT) reinforced composite thick skew plate where the CNTs reinforcements were aligned uniaxially in the axial direction. The first-order shear deformation theory and the element-free IMLS-Ritz method were used for theoretical formulation and numerical analysis, respectively. Finally, some parameters such as different boundary conditions, CNT ratios, skew plates, aspect ratios and thickness-to-height ratios were studied. Golmakani and Rezatalab [16] considered nonlocal Mindlin plate theory to analyze the non-uniform biaxial buckling behavior of orthotropic single-layered graphene sheet embedded in a Pasternak elastic medium. The first-order shear deformation theory was used to derive the generalized displacements. The governing equations for various boundary conditions were solved by differential quadrature method (DQM). Eventually, some parameters such as the effects of small scale, aspect ratio, polymer matrix properties and boundary conditions were investigated. Mao et al. [17] studied the creep buckling and post-buckling analysis of the laminated piezoelectric viscoelastic plates made of functionally graded materials (FGM) subjected to an in-plane compressive load. The elastic piezoelectric theory and Boltzmann superposition principle were used to derive nonlinear creep governing equations of the laminated piezoelectric viscoelastic FGM plates. Some parameters such as the effects of geometric nonlinearity, the applied electric load as well as the volume fraction and the geometric parameters on the creep buckling and post-buckling of the plate were investigated. Lei et al. [18] used the element-free kp-Ritz method to study dynamic stability of carbon nanotube-reinforced functionally graded cylindrical panels subjected to static and periodic axial force. Single-walled carbon nanotubes (SWCNTs) with different distributions were employed to reinforce the cylindrical panel. Effective materials properties of the nanocomposites were estimated using Eshelby–Mori–Tanaka approach. The Ritz minimization procedure was applied to the energy expression leading to formulate a provision of Mathieu–Hill equations. Bolotin's first approximation was then employed to analyze the principal instability regions. Finally, the effects of such parameters as volume fraction of carbon nanotubes, radius to thickness as well as edge to radius ratios, boundary condition and types of carbon nanotube distributions were studied. Zhang et al. [19] studied the flexural strength and free vibration of carbon nanotube reinforced composite cylindrical panels. Three types of functionally graded distributions and four kinds of distributions of uniaxially aligned reinforcements were considered. An equivalent continuum model based on the Eshelby–Mori–Tanaka approach was used to estimate the material properties of nanocomposite panel. The first-order shear deformation shell theory was employed based on which the governing equations were developed. The effect of such parameters as volume fraction of carbon nanotubes, thickness, edge-to-radius ratio, boundary condition and manner of distribution of carbon nanotube on flexural strength and free vibration responses of the panels were examined.

Despite the variety of research works carried out on the composite materials containing CNT or piezoelectric materials, there are only a few articles in the literature dealing with combined effects CNT and piezoelectric components on the mechanical behavior. Therefore, motivated by the above research works, the present article presents buckling behavior of piezoelectric composites panel reinforced with CNTs subjected to axial and biaxial loads.

2. Simulation

2.1. Mathematical modeling

A schematic figure of the panel is shown by Fig. 1. It is used for modeling CNT structures, that is, a thin-walled circular cylindrical shell of length L, wall thickness h, and undeformed-middle-surface radius R. h is also considered to be much smaller than R.

Cylindrical coordinates x, theta is used to be referred for the middle surface of the cylinder. The coordinate z, positive outward is also used to measure distance from the middle surface. u, v and w are used to denote displacement components in the x, θ, and z directions.

Displacement filed formulation is shown as below based on classical laminate theory [20]

$$u(x,\theta,z,t)=u(x,\theta,t)-z\frac{\partial w(x,\theta,t)}{\partial x},$$