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Free vibration analysis of a system of elastically interconnected rotating tapered Timoshenko beams using differential transform method

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ABSTRACT

A new procedure for determining natural frequencies and mode shapes of a system of elastically connected multiple rotating tapered beams is presented through a differential transform method. These identical double tapered beams are assumed to rotate at a constant speed and their deformation is obeying the Timoshenko beam theory. The motion of the system is described by a coupled set of $2n$ partial differential equations. A substantial change of variables is employed to uncouple the governing differential equations. Thereafter, a new equivalent system of n decoupled Timoshenko beams is formed where each beam appears elastically connected to the ground, resulting to a bunch of similar equations. The inverse transform is applied to extract the solution of the original system in terms of the original variables. The results are validated against those reported in the literature and then the effects of the rotational speed, hub radius, taper ratios, rotary inertia, shear deformation, slenderness ratio and elastic layers stiffness coefficients on the natural frequencies are discussed. The natural frequencies are in excellent agreement with the reported results.

1. Introduction

The determination of the dynamic characteristics of rotating beams is of great importance in the process of designing a multitude of engineering components such as turbine blades, compressor blades, propellers, helicopter rotors, long and flexible rotating space booms, robot manipulators and spinning space structures. In case the dimensions of the beam cross-section are comparable to that of the beam length or obtaining higher modes are required, adopting the Timoshenko beam theory that accounts for rotary inertia and shear deformation is recommended instead of Euler–Bernoulli's model.

Banerjee [1] developed the dynamic stiffness matrix of a centrifugally stiffened Timoshenko beam and carried out a free vibration analysis. Raffa and Vatta [2] investigated the Lagrangian formulation of a continuous, axisymmetric rotating Timoshenko beam. Du et al. [3] applied a convergent power series expression to obtain the natural frequencies and mode shapes of rotating Timoshenko beams. Wang et al. [4] extended Galerkin's method for rotating beam vibrations using Legendre Polynomials. The in-plane and out-of-plane free vibrations of a rotating Timoshenko beam are analyzed by means of a finite element technique by Yokoyama [5]. Lee and Kuo [6] studied the upper bound of the fundamental bending frequency of a rotating uniform Timoshenko beam with a general elastically restrained root through applying Rayleigh's principle. Curti et al. [7] proposed an analytical procedure, based on the dynamic stiffness method for studying rotor dynamics problems. Auciello and Ercolano [8] proposed a dynamic investigation method for the analysis of Timoshenko beams which takes into account the shearing deformation and the rotary inertia. Banerjee and Sobey [9] derived the mass and stiffness matrices of rotating twisted and tapered Timoshenko beams adopting standard finite element methods. Rao and Gupta [10] derived the stiffness and mass matrices of a rotating twisted and tapered beam element to find the natural frequencies and mode shapes of beams in the bending–bending mode of vibration. Bazoune et al. [11] adopted the finite element method to investigate the in-plane and out-of-plane modes of free vibration of a tapered Timoshenko beam mounted on the periphery of a rotating rigid hub. The dynamic stiffness method for the free vibration of a symmetric non-uniform Timoshenko beam is presented by Leung and Zhou [12]. Lee and Lin [13] studied bending vibrations of rotating non-uniform Timoshenko beams with an elastically restrained root. Rossi and Laura [14] investigated numerically the vibration of a linearly tapered Timoshenko beam. Yardimoglu [15] developed a new finite element model and used it for transverse vibrations of

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Nomenclature

A	cross-sectional area
b_0	beam breadth at the root section
b_L	beam breadth at point $x=L$
c_b	breadth taper ratio
c_h	height taper ratio
E	Young's modulus
h_0	beam height at the root section
h_L	beam height at point $x=L$
I_y	second moment of inertia about y-axis
k	normal stiffness coefficient per unit length of coupled system
$\overline{k}$	dimensionless normal stiffness coefficient per unit length of coupled system
k_θ	rotational stiffness coefficient per unit length of coupled system
$\overline{k}_\theta$	dimensionless rotational stiffness coefficient per unit length of coupled system
K	shear correction factor
KAG	shear rigidity
L	beam length
r	inverse of slenderness ratio
R	hub radius
s	shear deformation parameter
t	time
$T(x)$	centrifugal force
$\overline{T}$	dimensionless centrifugal force
x	longitudinal coordinate
y	flapwise bending displacement
α	normal stiffness coefficient per unit length of decoupled system
$\overline{\alpha}$	dimensionless normal stiffness coefficient per unit length of decoupled system
β	rotational stiffness coefficient per unit length of decoupled system
$\overline{\beta}$	dimensionless rotational stiffness coefficient per unit length of decoupled system
δ	dimensionless hub radius
θ	rotation angle due to bending
ρ	density of the beams material
ρA	mass per unit length
ω	circular natural frequency
$\overline{\omega}$	dimensionless natural frequency parameter
Ω	constant rotational speed
$\overline{\Omega}$	dimensionless rotational speed parameter

rotating tapered Timoshenko beams with rectangular cross-section. Zhu [16] derived the equations of motion by adopting an assumed mode method to investigate the free vibration of a rotating double-tapered cantilever Timoshenko beam subject to flapwise transverse excitations.

A multi-beam system, consisting of elastically connected multiple parallel beams, can exhibit important applications in several fields of civil and mechanical engineering. Kukla [17] studied the problem of free vibration of two axially loaded beams connected by translational springs. Oniszczuk [18] studied the free vibration of two parallel simply supported beams continuously joined by a Winkler elastic layer through the Euler–Bernoulli theory. The exact theoretical general solutions of undamped forced vibrations for a simply supported double-beam system are determined by Oniszczuk [19]. Vu et al. [20] presented a unique yet simple method of obtaining the exact solution for the forced vibration of a damped double Euler–Bernoulli beam system subject to harmonic excitation. Abu-Hilal [21] studied the dynamic response of a double-beam system traversed by a constant moving load. Stojanovic et al. [22] considered the free transverse vibration and buckling of a double-beam continuously joined by a Winkler elastic layer under compressive axial loading with the influence of rotary inertia and shear. Simsek and Cansiz [23] studied the dynamic responses of an elastically connected functionally graded double beam system carrying a harmonic load moving at constant speed. Stojanovic et al. [24] investigated a general procedure for the determination of the natural frequencies and buckling load for a set of beams under compressive axial loading modeled by Timoshenko and high-order shear deformation theory. In order to deal with the complexity caused by the coupling of the equations of motion in a multiple beams system, an original change of variables is applied by Ariaei et al. [25] to uncouple the equations of motion, making them efficiently solvable for different static/dynamic moving loads.

Partial differential equations are often used to describe engineering problems, the analytical solutions of which are often difficult to establish. As a result, approximate numerical methods such as finite element, finite difference, and boundary element are ultimately preferred [26]. However, in spite of the advantages of these available methods and their relevant general computer codes, the analytical and semi-analytical solutions are more appropriate due to their implication in the physics of the problem and their convenience in parametric studies. Considering the advantages of semi-analytical solutions, the differential transform method, DTM, is adopted in this study. In the available literature, there are several studies that have adopted DTM to deal with linear and nonlinear initial value problems, eigenvalue problems, ordinary and partial differential equations, etc. The DTM was first introduced by Zhou [27] in solving linear and nonlinear initial value problems in electrical circuit analysis. Chen and Ju [28] adopted DTM to predict the advective–dispersive transport problems. Arikoglu and Ozkol [29] extended DTM to solve the integro-differential equations. Catal [30] performed analysis of free vibration of beams on elastic soil adopting DTM. Ozdemir Ozgumus and Kaya [26] studied free vibration analysis of a rotating, double tapered Timoshenko beam featuring coupling between flapwise bending and torsional vibrations. Free vibration analysis of a rotating Timoshenko beam by applying DTM is investigated by Kaya [31]. Ozdemir Ozgumus and Kaya [32,33] performed free vibration analysis of a rotating tapered and double-tapered Timoshenko beam subject to flapwise bending vibration by adopting DTM. Rajasekaran [34] studied the free bending vibration of rotating axially functionally graded (FG) Timoshenko tapered beams with different boundary conditions adopting differential transform method and differential quadrature element method of lowest order. In addition to the variety of the cases where DTM can be applied, its simplicity and accuracy in calculating the natural frequencies and plotting the mode shapes makes it outstanding among many other methods.

All the referenced articles have considered one rotating tapered beam or a system of elastically connected non-rotating beams with a constant cross section. The contribution of this article is to consider a multiple tapered beams system rotating at a constant rotational speed. Considering n elastically connected beams, this proposed method involves a change of variables enabling the separation of a set of $2n$ differential equations into independent equations and to extend the previous work of the author [25] where non-rotating beams with constant cross section are