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Nonconservative stability of viscoelastic rectangular plates with free edges under uniformly distributed follower force

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ABSTRACT

Dynamic stability of viscoelastic rectangular plates under a uniformly distributed tangential follower load is studied. Two sets of boundary conditions are considered, namely, clamped in one boundary and free in other boundaries (CFFF) and two opposite edges simply supported and other two edges free (SFSF). By considering the Kelvin–Voigt model of viscoelasticity, the equation of motion of the plate is derived. The differential quadrature method is employed to obtain the numerical solution and it is verified against known results in the literature. Numerical results are given for the real and imaginary parts of the eigenfrequencies to investigate the divergence and flutter instabilities. It is observed that the type of stability differs for CFFF and SFSF plates indicating the strong influence of the boundary conditions on the dynamic stability of viscoelastic plates. In particular it is found that CFFF plates undergo flutter instability and SFSF plates divergence instability. One consequence is that SFSF plates become unstable at a load less than the load for CFFF plates as the effects of viscoelasticity as well as the aspect ratio are found to be minor for SFSF plates.

1. Introduction

The dynamic stability of systems such as beams, plates, shells, pipes conveying fluid and rockets subject to follower forces has been studied extensively. Plate structures are of importance in diverse fields of technology like aeronautics, automotive design and offshore structures, and as a result substantial work has been performed on their stability under nonconservative loads. It has been observed by Herrmann [1] that the load parameter has a great effect on the stability of an elastic system subjected to a nonconservative force. By considering a cantilever plate subjected to biaxial subtangential loading, Farshad [2] studied the effect of load parameter on dynamic stability. Influence of aspect ratio on the stability of a plate subjected to conservative and non-conservative forces was studied by Adali [3]. Various effects on dynamic stability of rectangular plates have been investigated in Leipholz and Pfent [4], Kumar and Srivasta [5], Higuchi and Dowell [6], Zuo and Schreyer [7], Kumar et al. [8], Kim and Park [9], Kim and Kim [10] and in Jayaraman and Strusthers [11].

More recently dynamic stability of viscoelastic structures has been the focus of a number of publications. Stability of viscoelastic columns under follower forces has been studied by Langthjem and Sugiyama [12], Darabseh and Genin [13] and Zhuo and Fen [14]. The corresponding work for viscoelastic plates is given in Eshmatov [15] for follower forces, in Wang et al. [16], Wang and Zhou [17] for uniformly tangential and in Robinson and Adali [18] for triangularly distributed tangential follower forces. Robinson [19] took non-linearity and tangential follower forces into account for simply supported plates, and Wang et al. [20] the effect of piezoelectric layers for viscoelastic plates with a combination of simple and clamped supports. Despite the increasing attention on the stability of viscoelastic plates subject to follower forces, the boundary conditions which appeared in the literature so far include only the clamped and simply supported cases [16,17,19,20]. It is noted that the main difference in the nonconservative stability of viscoelastic columns and plates is that the formulations for the two-dimensional structures lead to governing equations expressed in the complex domain leading to complex eigenvalue problems.

A rectangular plate may experience divergence or flutter instability depending on the boundary conditions and quite often plates with free boundaries are employed in practice. In the present study, the stability of rectangular viscoelastic plates subject to a uniformly distributed tangential follower force and free boundary conditions is studied using the Kelvin–Voigt model of viscoelastic behavior. In particular dynamic stability of viscoelastic plates with CFFF and SFSF boundary conditions is established where C, F and S stand for clamped, free and simply supported boundary conditions, respectively. Free boundary conditions are experienced in many engineering applications indicating the importance of studying the dynamic stability for these cases. In the present

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study, differential quadrature method [19,21] is employed to solve the governing equation which is expressed in the complex domain using Laplace transformation. Previously the differential quadrature method was applied to nonconservative stability in Marzani et al. [22].

In Section 2, the equations governing the vibrations of nonconservatively loaded viscoelastic plates are established using Laplace transformation following the approach implemented in Wang et al. [16], Wang and Zhou [17] and Wang et al. [23]. In Section 3 the differential quadrature method is implemented to discretize the equation of motion and the boundary conditions. This is followed by the verification of results in Section 4 and numerical results in Section 5. Numerical results are given to investigate the divergence and flutter instabilities for CFFF and SFSF plates by way of plotting the real and imaginary parts of the eigenvalues with respect to the follower load. The effects of the aspect ratio and viscoelastic constant on stability are also studied. Finally, Section 6 is devoted to concluding remarks.

2. Equation of motion for viscoelastic plate

We consider a thin rectangular plate of dimensions $a \times b$ and thickness h with Young's modulus of E, Poisson's ratio ν and density ρ. The Cartesian coordinate system x, y, z which has its origin at mid-thickness is shown in Fig. 1. Using the Kirchhoff plate theory, the displacements u, v, w along x, y and z directions, respectively, are given by

$$u = -z\psi_x, \quad v = -z\psi_y, \quad w = w(x, y, t) \tag{1}$$

where the angles of rotation ψ_x and ψ_y are related to the transverse displacement w through the relations

$$\psi_x = \frac{\partial w}{\partial x}, \quad \psi_y = \frac{\partial w}{\partial y} \tag{2}$$

The linear strain–displacement relations are given by

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2}, \quad \varepsilon_{xy} = \frac{\gamma_{xy}}{2} = -z\frac{\partial^2 w}{\partial x \partial y} \tag{3}$$

where ε_x and ε_y are the normal strain components, and γ_{xy} is the shear strain component.

In the present study the plate material is taken as viscoelastic of the Kelvin–Voigt type. The constitutive equations for this case can be written as in Refs. [16–18, 20].

$$\mathbf{s}_{ij} = 2G\ \mathbf{e}_{ij} + 2\ \eta\ \dot{\mathbf{e}}_{ij} \tag{4a}$$

$$\sigma_{ii} = 3K\varepsilon_{ii} \tag{4b}$$

where K, η, G are bulk modulus, viscoelastic coefficient and shear modulus, respectively. They can be expressed as $K = E/3(1-2\nu)$ and $G = E/(1+2\nu)$ in terms of E and ν. The quantities $\mathbf{s}_{ij}$ and $\mathbf{e}_{ij}$ are, respectively, the deviatoric tensors of stress and strain while $\mathbf{s}_{ii}$ and σ_{ii} stand for the spherical tensors of strain and stress. The bending moments M_x, M_y and twisting moments M_{xy}, M_{yx} are given by:

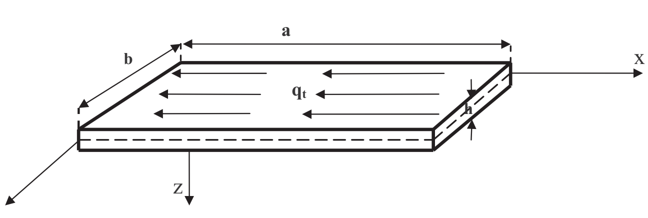

Fig. 1. Viscoelastic plate subject to distributed tangential follower force q_t.

$$M_x = \int_{-h/2}^{h/2} z\sigma_x dz, \quad M_y = \int_{-h/2}^{h/2} z\sigma_y dz \tag{5a}$$

$$M_{xy} = \int_{-h/2}^{h/2} z\sigma_{xy} dz, \quad M_{yx} = \int_{-h/2}^{h/2} z\sigma_{yx} dz \tag{5b}$$

where σ_x and σ_y are the normal stress components, σ_{xy} and σ_{yx} are the shear stress components. The plate is subject to a uniformly distributed tangential follower force q_t as shown in Fig. 1. The equation governing the vibrations of the plate under the distributed follower force can be written as

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - q_t(a-x)\frac{\partial^2 w}{\partial x^2} - \rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{6}$$

Following the methodology employed in [16] and [17], Laplace transformations of Eqs. (4)–(6) are performed. Carrying out the inverse Laplace transformations of the resulting equations [24], the differential equation governing the vibration of the nonconservative viscoelastic rectangular plate is obtained as

$$\frac{h^3}{12}\left(A_3 + A_4\frac{\partial}{\partial t} + A_5\frac{\partial^2}{\partial t^2}\right)\nabla^4 w + q_t(a-x)\left(A_1 + A_2\frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial x^2} + \left(A_1 + A_2\frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial t^2} = 0 \tag{7}$$

where

$$A_1 = 3K + 4G, \quad A_2 = 4\eta, \quad A_3 = 4G(3K+G), \quad A_4 = 4\eta(2G+3K), \quad A_5 = 4\eta^2 \tag{8}$$

and

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \tag{9}$$

Introducing the dimensionless variables

$$X = \frac{x}{a}, \quad Y = \frac{y}{b}, \quad \overline{w} = \frac{w}{h}, \quad \lambda = \frac{a}{b} \tag{10a}$$

$$q = \frac{12 q_t a^3 (1-\nu^2)}{E h^3}, \quad \tau = \frac{th}{a^2}\sqrt{\frac{E}{12\rho(1-\nu^2)}}, \quad H = \frac{\eta h}{a^2 E}\sqrt{\frac{E}{12\rho(1-\nu^2)}} \tag{10b}$$

the governing Eq. (7) can be rewritten as

$$\left(1 + c_1\frac{\partial}{\partial \tau} + c_2\frac{\partial^2}{\partial \tau^2}\right)\nabla^4\overline{w} + q(1-X)\left(1 + c_3\frac{\partial}{\partial \tau}\right)\frac{\partial^2 \overline{w}}{\partial X^2} + \left(1 + c_3\frac{\partial}{\partial \tau}\right)\frac{\partial^2 \overline{w}}{\partial \tau^2} = 0 \tag{11}$$

where τ is dimensionless time, H is dimensionless delay time of the material, and

$$c_1 = \frac{4(2-\nu)(1+\nu)}{3}H, \quad c_2 = \frac{4(1-2\nu)(1+\nu)^2}{3}H^2, \quad c_3 = \frac{4(1-2\nu)(1+\nu)}{3(1-\nu)}H \tag{12}$$

are real constants which depend on the delay time H, and

$$\nabla^4\overline{w} = \frac{\partial^4 \overline{w}}{\partial X^4} + 2\lambda^2\frac{\partial^4 \overline{w}}{\partial X^2 \partial Y^2} + \lambda^4\frac{\partial^4 \overline{w}}{\partial Y^4} \tag{13}$$

The solution of Eq. (11) is expressed in the form

$$\overline{w}(X, Y, \tau) = W(X, Y)\exp(j\omega\tau) \tag{14}$$

where $j = \sqrt{-1}$ and ω is the dimensionless frequency which is in general a complex number. Substituting Eq. (14) into Eq. (11), one obtains the differential equation

$$\left(1 + c_1 j\omega + c_2 j^2\omega^2\right)\nabla^4 W + q(1-X)(1 + c_3 j\omega)\frac{\partial^2 W}{\partial X^2} + (1 + c_3 j\omega) j^2\omega^2 = 0 \tag{15}$$