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Size-dependent axial buckling and postbuckling characteristics of cylindrical nanoshells in different temperatures

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ABSTRACT

One of the most important reasons that give rise to the extraordinary behaviors of nanostructures is the free surface energy. In the current investigation, a size-dependent shell model is introduced which has an excellent capability to take surface energy effects into account. To this end, Gurtin–Murdoch elasticity theory is implemented into the classical shell theory. Using virtual work's principle, the non-classical governing differential equations related to the cylindrical nanoshell subjected to axial compressive load are derived. Subsequently, a boundary layer theory is extended to solve the problem with considering the effects of surface free energy in addition to the nonlinear prebuckling deformations and the large postbuckling deflections. Finally, a solution methodology based on a two-stepped perturbation technique is put to use in order to obtain the size-dependent critical buckling loads and related postbuckling equilibrium paths corresponding to different surface properties and various sets of thermal environments. It is found that for all sets of thermal environment, the surface free energy has significant influence on the postbuckling strength of nanoshell. Also, it is seen that thermal environment causes to decrease the both critical buckling load and critical end-shortening, but it has a negligible influence on the value of minimum postbuckling load.

1. Introduction

Nanostructures such as nanobeams, ultra thin plates (nanofilms), nanoshells and etc have attracted much attention from the scientific community due to their considerable enormous physical, mechanical, and electrical properties [1–4]. Nanoshells are one of the gifts by nanotechnology for various applications. For example, nanoshell biosensors work by emitting a signal that is characteristic of the virus, toxin, or bacterial to be measured, thus identifying the presence or absence of the material [5]. To achieve more effective and better diagnostic and/or therapeutic goals, nanoshells can be conjugated to antibodies, oligonucleotides fluorophores, targeting ligands, polymers, therapeutic agents, and radioisotope [6].

At this submicrion size, the behavior of structures affected by various parameters known as size-effects. Surface stress effect is one of these effects through high surface to bulk ratio of nanostructures that causes to exhibit different behaviors compared to the conventional structures. There are different investigations in which the size-dependant responses of various types of nanostructures have been studied [7–10].

The surface energy effect can be easily observed at the atomic scale, and this object has been clearly indicated and explained [11–13]. This phenomenon can be explained as the atoms at or near a free surface have different equilibrium requirements than the atoms have in the bulk of the material because of different environment conditions. This difference causes excess surface energy as a superficial energy term since a surface can be interpreted as a layer to which certain energy is attached [14].

Modified continuum models have been the subject of much attention in nanomechanics due to their computational efficiency and lesser complexity which can produce accurate results comparable to the atomistic models ones [15–29]. Gurtin and Murdoch [30,31] developed a theoretical framework based on the continuum mechanics including the effects of surface free energy, in which the surface is simulated as a mathematical layer of zero thickness with different material properties for the underlying bulk which is completely bonded by the membrane. Gurtin–Murdoch elasticity theory has the capability to incorporate the effects of surface free energy into the mechanical response of nanostrustures which has been applied in many studies conducted for various problems about mechanical behavior of the structures at nanoscale.

Wang and Feng [32] and Abbasion et al. [33] investigated the free vibration response of microsclae beam including surface effects on the basis of Euler–Bernoulli and Timoshenko beam

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theories, respectively. Tian and Rajapakse [34] implemented the surface elasticity theory to examine the surface-interface stress effects on the elastic field of an isotropic matrix with a nanoscale elliptical inhomogeneity. Lu et al. [35] presented a generalized refined theory incorporating the influence of surface stress for functionally graded films based on Gurtin–Murdoch elasticity theory. Zhao and Rajapakse [36] proposed the axisymmetric solutions for an elastic layer subjected to surface loading incorporating the effects of surface energy. Fu et al. [37] studied the influences of surface energy on the free vibration and buckling behavior of nanobeams in the both linear and nonlinear regimes using Galerkin's technique. Ansari and Sahmani [38] analyzed the bending and buckling behaviors of nanobeams including the effect of surface stress corresponding to different beam theories. Ansari and Sahmani [39] also predicted the free vibration of rectangular nanoplates based on surface elasticity theory. They implemented Gurtin–Murdoch elasticity theory into the classical and first-order shear deformation plate theories. Ansari et al. [40,41] predicted the postbuckling characteristics of Euler–Bernoulli and Timoshenko nanobeams, respectively, with the presence of the surface stress effects. On the basis of surface elasticity theory, Mouloodi et al. [42] carried out the finite element implementation of free vibration response of nanoplates considering surface stress effects. Sahmani et al. [43] predicted the free vibration response of postbuckled third-order shear deformable nanobeams based on surface elasticity theory. Sahmani et al. [44] used Gurtin–Murdoch elasticity theory to develop a non-classical beam model to study the nonlinear forced vibrations of nanobeams including surface effects. Sahmani et al. [45] examined the free vibration of postbuckled circular higher-order shear deformable nanoplates incorporating the effect of surface free energy. Recently, Mohebshahedin and Farrokhabadi [46] demonstrated the effects of surface free energy on the instability of NEMS tweezers and cantilevers fabricated from conductive cylindrical nano-wires. Sahmani et al. [47] studied the free vibration characteristics of postbuckled functionally graded third-order shear deformable nanobeams using surface elasticity theory.

In the present investigation, the nonlinear buckling and postbuckling behavior of axially compressed cylindrical nanoshells subjected to different thermal environments is studied including the effects of surface free energy. An efficient size-dependent shell model is developed based on Gurtin–Murdoch elasticity theory. A boundary layer theory is employed considering simultaneously the surface energy effects, the nonlinear prebuckling deformations and the large postbuckling. At the end, the critical buckling loads and related equilibrium postbuckling paths are obtained using a two-stepped singular perturbation technique.

2. Preliminaries

A cylindrical nanoshell with the length L, thickness h, and mid-surface radius R is considered as depicted in Fig. 1. The nanoshell includes a bulk part and two additional thin surface layers (inner and outer layers). For the bulk part, the material properties are Young's modulus E and Poisson's ratio ν. The two surface layers are assumed to have surface elasticity modulus of E_s, Poisson's ratio ν_s and the surface residual tension τ_s. According to a curvilinear coordinate system with its origin located on the middle surface of nanoshell, coordinates of a typical point in the axial, circumferential and radial directions are denoted by x, y and z, respectively. Now, based on the classical shell theory, the displacement field can be expressed as

$$u_x(x,y,z) = u(x,y) - z\frac{\partial w(x,y)}{\partial x} \tag{1a}$$

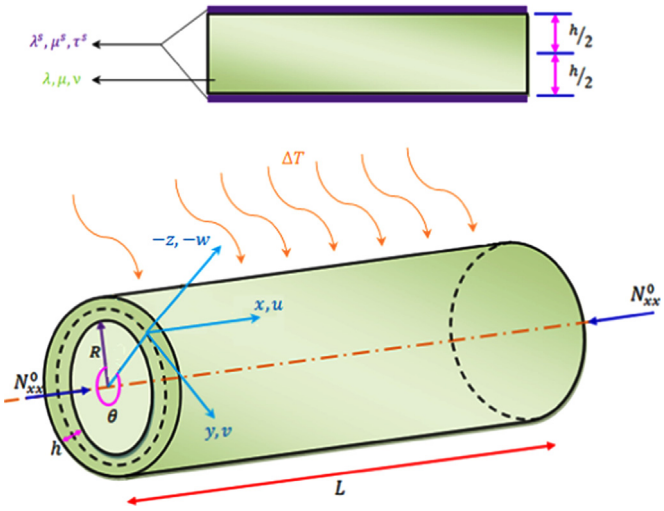

Fig. 1. Schematic view of a cylindrical nanoshell with surface layers.

$$u_y(x,y,z) = v(x,y) - z\frac{\partial w(x,y)}{\partial y} \tag{1b}$$

$$u_z(x,y,z) = w(x,y) \tag{1c}$$

in which u, v and w represent the middle surface displacements

Following the von Karman–Donnell-type kinematics of nonlinearity [48], which is on the basis of this stipulation that the thickness of the shell h, is remarkably small in comparison with its radius of curvature R, the kinematical strain–displacement relationships can be expressed as below

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon^M_{xx} \\ \epsilon^M_{yy} \\ \gamma^M_{xy} \end{Bmatrix} + \begin{Bmatrix} \epsilon^T_{xx} \\ \epsilon^T_{yy} \\ \gamma^T_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon^0_{xx} \\ \epsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z\begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} + \begin{Bmatrix} \alpha\Delta T \\ \alpha\Delta T \\ 0 \end{Bmatrix}$$
$$= \begin{Bmatrix} \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \end{Bmatrix} - z\begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} + \begin{Bmatrix} \alpha\Delta T \\ \alpha\Delta T \\ 0 \end{Bmatrix} \tag{2}$$

where $\epsilon^M_{xx}, \epsilon^M_{xx}, \gamma^M_{xy}$ and $\epsilon^T_{xx}, \epsilon^T_{xx}, \gamma^T_{xy}$ stand for mechanical and thermal strain components, respectively. Also, $\epsilon^0_{xx}, \epsilon^0_{xx}, \gamma^0_{xy}$ denote the strain components of the middle surface, $\kappa_{xx}, \kappa_{yy}, \kappa_{xy}$ are the curvature components of nanoshell, α and ΔT represent the thermal expansion coefficient and temperature change, respectively.

Moreover, the constitutive relations are as follow

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} \begin{Bmatrix} \epsilon^M_{xx} \\ \epsilon^M_{yy} \\ \gamma^M_{xy} \end{Bmatrix}$$
$$- \begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \alpha\Delta T \\ \alpha\Delta T \end{Bmatrix} \tag{3}$$

in which $\lambda = \frac{E\nu}{(1-\nu^2)}$, $\mu = \frac{E}{(2(1+\nu))}$ are Lame's constants.

The classical models of continuum mechanics do not have the capability to consider the atomic features of the nanostructures. However, according to the simplicity and computational efficiency of continuum mechanics, different modified continuum models have been developed to incorporate size-effects into the conventional continuum approach. Gurtin–Murdoch continuum elasticity is one of the most efficient theories to incorporate surface stress effect into analysis of mechanical behaviors of structures at nanoscale. According to this non-classical continuum theory, the