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Analysis of shape variation during hydro-forming of ellipsoidal shells with double generating lines

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ABSTRACT

In order to obtain an ideal ellipsoidal shell by the controlling of inputting pressured water volume, the curvature and volume variation have been analyzed for hydro-forming of ellipsoidal shell with double generating lines. The geometric criteria of double generating lines changing into single generating line was derived, and the mathematical model between shell volume and axis length ratio was developed. It is shown that the volume variation is only associated with the initial and final axial length ratios, and can be theoretically predicted. An experiment was conducted on hydro-forming of the ellipsoidal shells with the axis length ratio 1.5, 1.7 and 2.2, respectively, and the effect of internal pressure was discussed on the deformation of the shell shape, curvature and volume variation. It is experimentally shown that the short axis is linearly increased, the long axis is slightly changed and the shell volume is exponentially increased. It is proved that the theoretical prediction of the volume variation has a good agreement with the experimental results, which indicates that the needed ideal ellipsoidal shell can be successfully formed by controlled of inputting pressured water volume.

1. Introduction

Ellipsoidal shells have been usually used as the water tanks and bottom of fuel tanks in the rockets due to the advantages of the small wind area, lower centroid and space saving. Since the die-less hydro-forming technology of closed shells was introduced [1], it was widely used to manufacture ellipsoidal shell with various sizes and materials. The experiments were early dealt with for the hydro-forming of ellipsoidal shells with axis length ratios 1.88, 1.67 and 1.25, and the influence of the axis length ratios, lateral petal numbers and materials were deeply discussed on the plastic deformation and wrinkling behavior [2]. It is experimentally proved that the wrinkling occurs in the equatorial plane for the ellipsoidal shells with axis length ratio 1.88 and 1.67 respectively, but it is well formed for the ellipsoidal shell with axis length ratio 1.25.

More comprehensive theoretical analysis was introduced on the hydro-forming of ellipsoidal shells [3,4]. It is pointed out that the axis length ratio $\sqrt{2}$ is a critical value. When the axis length ratio exceeds $\sqrt{2}$,there is a latitudinal compressive stress in the equatorial plane, which will lead to wrinkling occurrence if the value of latitudinal compressive stress is beyond the critical wrinkling stress. In order to avoid the latitudinal compressive stress, an ellipsoidal shell with double generating lines was designed as the pre-form shell [5]. In this design, the ellipsoidal shell segment which has strong possibility of wrinkling occurrence can be replaced by another ellipsoidal shell segment with axis length ratio less than $\sqrt{2}$. Experiments were subsequently carried out for hydro-forming of ellipsoidal shells with double generating lines, and the variation of stress state and shell shape were analyzed. It is shown that a sound ellipsoidal is obtained, and the wrinkling is successfully avoided. Similarly, this idea can be also used to resolve the problem that the polar plate is not deformed during hydro-form of prolate ellipsoidal shell [6].

Stress analysis on the ellipsoidal shell with double generating lines subjected to internal pressure was theoretically carried out, and the initial yield pressure was derived [7]. Stress locus of typical points was experimentally illustrated, and the expanding of plastic deformation was analyzed. It is shown that the plastic deformation first occurs on the pole and then expands towards to equatorial line, and the sheet materials are deformed prior to the weld seams along latitudinal direction. The initial yield pressure of the shell obtained by experiment is well agreed with the theoretical prediction.

In order to reverse an optimum pre-form structure according to needed ellipsoidal shell, a mathematical response model between axis lengths and structural parameters was developed on the basis of the Box–Behnken design method [8]. For the needed ellipsoidal

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Nomenclature

r	First principal curvature radius
R	Second principal curvature radius
ϕ	Rotating angle
a	Long axis
b	Short axis
λ	Axis length ratio
p	Internal pressure
t	Thickness
σ_φ	Longitudinal stress
σ_θ	Latitudinal stress
α	Theoretical dividing globe angle
α_0	Dividing globe angle
a_1	Long axis of first ellipsoidal shell
a_2	Long axis of second ellipsoidal shell
b_1	Short axis of first ellipsoidal shell
b_2	Short axis of second ellipsoidal shell
λ_1	Axis length ratio of first ellipsoidal shell
λ_2	Axis length ratio of second ellipsoidal shell
λ_F	Axis length ratio of final ellipsoidal shell
ϕ_1	Rotating angle of first ellipsoidal shell
ϕ_2	Rotating angle of second ellipsoidal shell
V	Final volume
V_0	Initial volume
(x_0, y_0)	Coordinate on dividing globe angle

shell with final axis length ratio $\lambda = 1.5$, the optimum initial axis length ratio of pre-form shell is $\lambda = 2.0 \sim 2.2$. By comparison with the experimental results, it is indicated that the predicted axis lengths calculated by reverse design are in good agreement with the experimental data, and the deviation is no more than 1%. However the forming pressure during optimization deeply depend on the materials and axis length ratio, the optimum pre-form shell is not generally applied to any situation.

The materials of ellipsoidal shells widely used in engineering manufacture are always stainless steel, low carbon steel and low alloy steel, and the maximum diameter is up to 3 m. For the ellipsoidal shells with various materials and initial axis length ratios, the forming pressure and axis lengths variation are still different. In order to obtain the needed ellipsoidal shell, it is essential to evaluate the forming pressure, constantly record the axis lengths and roundness during hydro-forming. Hence, it is a time consuming process.

To obtain the needed ideal ellipsoidal shell with single generating line and realize shape controlling by inputting pressured water volume, a mathematical model between volume flow and axis length ratio was developed by theoretical analysis. An experiment was carried out and the effect of internal pressure was discussed on shell shape, axis lengths, curvature and volume variation. Finally, the sound ellipsoidal shell is obtained.

2. Theoretical analysis

2.1. Design of ellipsoidal shell with double generating lines

Fig. 1 shows the geometry of an ellipsoidal shell subjected to the internal pressure, where r is the first principal curvature radius, R is the second principal curvature radius, ϕ is rotating angle between revolution axis y and second principal curvature radius R, a is radius of the long axis and b is radius of the short axis. The axis length ratio is defined as $\lambda = a/b$. The general geometric relationship is expressed in Eq. (1).

$$r = \lambda a k^3 \quad R = \lambda a k \quad k = \frac{1}{\sqrt{(\lambda^2 - 1)\sin^2\phi + 1}} \tag{1}$$

When the ellipsoidal shell is subjected to the internal pressure, according to Laplace equation, the longitudinal stress σ_φ and the latitudinal stress σ_θ can be expressed as follows [3]:

$$\sigma_\varphi = \frac{p}{2t}R, \ \sigma_\theta = \frac{pR}{2t}\left(2 - \frac{R}{r}\right) \tag{2}$$

where, p is the internal pressure; t is the thickness.

For the ellipsoidal shell with initial axis length ratio $\lambda > \sqrt{2}$, there is a theoretical globe dividing angle α on the ellipsoidal shell, where the latitudinal stress varies from tensile state to compressive state. In the previous research [5], the relationship between theoretical globe dividing angle α and axis length ratio λ has been given as follows:

$$\alpha = arc\ \tan \frac{\sqrt{\lambda^2 - 2}}{\lambda^2} \tag{3}$$

Fig. 2 shows the variation of theoretical globe dividing angle α with the increase of initial axis length ratio λ. There is a peak value of theoretical globe dividing angle α.

Derivative operation on Eq. (3), it is expressed as follows:

$$\alpha' = f'(\lambda) = \frac{\lambda(4 - \lambda^2)}{(\lambda^4 + \lambda^2 - 2)\sqrt{\lambda^2 - 2}} \tag{4}$$

For $\alpha' = 0$, it is shown that when the initial axis length ratio is $\lambda = 2.0$, the maximum value of theoretical globe dividing globe angle is $\alpha_{max} = 19.5°$.

Based on the mechanical analysis mentioned above, it is theoretically proved that there is a strong possibility of wrinkling occurrence between dividing line and equatorial line. Fig. 3 shows the geometric structure of the ellipsoidal shell with double generating lines, which is composed of two ellipsoidal shells with different axis length ratios. The axis length ratio of the first ellipsoidal shell is $\lambda_1 > \sqrt{2}$, and that of the second ellipsoidal shell is $\lambda_2 < \sqrt{2}$. The initial axis length ratio of the ellipsoidal shell with double generating lines is defined as $\overline{\lambda} = a_2/b_1$, where a_2 is the length of the long axis and b_1 is the length of the short axis. In order to obtain double tensile stress state for any position of ellipsoidal shell, the dividing globe angle is $\alpha_0 > \alpha_{max}$.

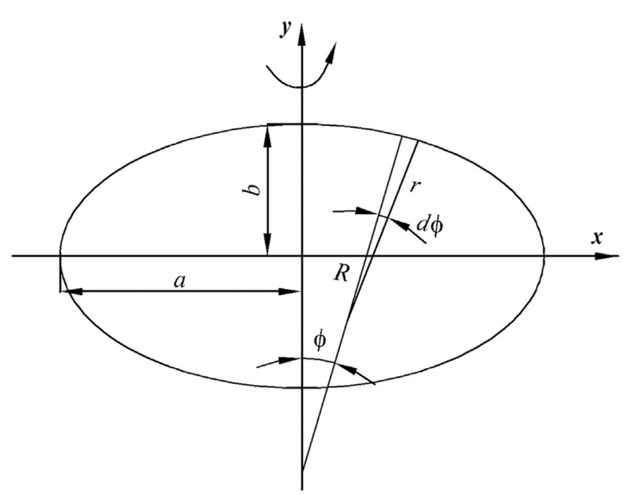

Fig. 1. Ellipsoidal shell revolving along the y-axis.