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Postbuckling of pressure-loaded nanotube-reinforced composite doubly curved panels resting on elastic foundations in thermal environments

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ABSTRACT

This paper presents an investigation on the postbuckling behavior of doubly curved nanocomposite panels reinforced by carbon nanotubes (CNTs) subjected to lateral pressure. The functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) are assumed to have CNTs linearly graded in the thickness direction. The overall mechanical properties of the FG-CNTRCs, which include the thermal effect of CNTs and the matrix, are estimated through a micromechanical model. The panels may rest on elastic foundations. The governing differential equations for the doubly curved panels are based on a higher order shear deformation shell theory with von Kármán strain–displacement relationships and the panel–foundation interaction. The initial deflections caused by lateral pressure and thermal bending stresses are both taken into account. The governing equations are further deduced to a boundary layer type problem that includes nonlinear prebuckling deformations and initial geometric imperfections of the panels which are subsequently solved using a two-step perturbation approach. The influences of CNT volume fraction, temperature variation, panel geometric parameters, as well as foundation stiffness on the postbuckling behavior of FG-CNTRC doubly curved panels are investigated.

1. Introduction

Doubly curved composite panels under external pressure are frequently encountered in engineering practice and their stability behavior is an important issue in engineering analysis and design. One of the problems deserving special attention is the study of the postbuckling of doubly curved panels subjected to mechanical loads in thermal environments.

Finding postbuckling solutions for a curved panel subjected to lateral pressure is a more difficult task than the case of the same panel subjected to axial compression. This is because the curved panel will firstly bend under lateral pressure before the buckling occurs. It has been reported that the postbuckling behaviors of plates, shells and panels are different. Librescu and his co-authors [1,2] studied postbuckling of composite laminated and sandwich doubly curved panels under combined mechanical and thermal loadings. They concluded that the postbuckling equilibrium path of curved panels is no longer the bifurcation type when the panels are subjected to lateral pressure. van Campen and his co-authors [3,4] studied postbuckling of orthotropic doubly curved panels subjected to lateral uniform pressure by using the adjacent equilibrium method based on the classical thin shell theory. They found that the postbuckling equilibrium path of curved panels is composed of two curves where the first curve is the bending curve and the second curve is the postbuckling curve and they defined the transition point as the second bifurcation point. Duc and his co-authors [5,6] studied postbuckling of functionally graded ceramic-metal FGM doubly curved panels resting on elastic foundations subjected to thermomechanical loads by using the Galerkin method. They observed that the postbuckling equilibrium path is a snap-through type for both thin and shear deformable FGM spherical panels.

Utilizing the concept of functionally graded (FG) materials, a new class of emerging composite materials, that is the FG carbon nanotube-reinforced composite (CNTRC), has been proposed by making use of CNTs as the reinforcements in a functionally graded pattern. One of the potential applications of CNTRCs is in Micro-Electro-Mechanical Systems (MEMS) and Nano-Electro-Mechanical Systems (NEMS) where CNTRCs can be used as key structural

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components [7,8]. The major difference between the conventional carbon fiber-reinforced composites and the carbon nanotube-reinforced composites lies in that the former can contain very high percentage of the carbon fibers (usually over 60% by volume), while the latter only has a low percentage of CNTs (about 2–5% by weight) [9–12]. This is due to the fact that a larger CNT volume fraction in CNTRCs can actually lead to the deterioration of the mechanical properties of the composites [13]. The functionally graded distribution of CNTs in CNTRCs has the advantage to arrange the CNTs at the locations with the most effective reinforcements.

The behavior of FG-CNTRC structures was first studied by Shen [14]. He considered the composite with CNT distributions within an isotropic matrix designed specifically to grade them with certain rules along the desired directions to improve the mechanical properties of the composite. After this initial work a series of investigations on FG-CNTRC beams [15–18], plates [19–27] and shells [28–34] were conducted to study the buckling behaviors of the FG-CNTRC structures. The buckling and postbuckling studies on CNTRC cylindrical panels, however, are relative scarce. Zhang et al. [35] studied the large deflection of FG-CNTRC cylindrical panels subjected to lateral uniform pressure and/or point load using the element-free kp-Ritz method. In their analysis, the material properties of functionally graded CNTRCs are assumed to be graded in the thickness direction and are estimated through an equivalent continuum model based on the Eshelby–Mori–Tanaka approach. Liew et al. [36] carried out studies on the compressive postbuckling of FG-CNTRC cylindrical panels subjected to axial compression by employing the element-free kp-Ritz method based on the first order shear deformation shell theory. More recently, based on a higher-order shear deformation shell theory Shen and Xiang [37–40] studied the nonlinear bending and compressive and thermal postbuckling of FG-CNTRC cylindrical panels resting on elastic foundations in thermal environments. It is noted that for all the aforementioned studies [35–40], only Shen and Xiang [37–40] took consideration of the effective material properties of CNTRCs being temperature-dependent. To the best of the authors' knowledge, there is no literature covering the nonlinear response of CNTRC doubly curved panels subjected to lateral pressure and resting on elastic foundations in thermal environments.

In the present work, we focus our attention on the postbuckling of CNTRC doubly curved panels subjected to lateral pressure in thermal environments. Two types of CNT distributions in the doubly curved panels, i.e. (i) uniformly distributed (UD) and (ii) functionally graded distributions, are considered. The material properties of CNTRCs are assumed to be temperature-dependent. A micromechanical model is employed to obtain the material properties of the CNTRC panels. The panels may rest on elastic foundations. The governing equations for the CNTRC doubly curved panels are derived based on a higher order shear deformation theory and von Kármán strain–displacement relationships and the panel–foundation interaction and thermal effects are also included in the governing equations. The boundary conditions are assumed to be either simply supported or clamped. The governing equations are first deduced to a boundary layer type that includes nonlinear prebuckling deformations and initial geometric imperfections of the panels. These equations are then solved by means of a singular perturbation technique along with a two-step perturbation approach to determine the postbuckling equilibrium paths for the CNTRC doubly curved panels. The novelty of this study results from accounting for the initial deflections caused by both the lateral pressure and the thermal bending stresses in the postbuckling analysis of CNTRC doubly curved panels.

2. Theoretical development

Fig. 1 shows a CNTRC doubly curved panel of rectangular planform resting on an elastic foundation. The total thickness of the panel, the length in the X and Y directions and the two radii of curvature to the middle surface of the panel are designated by h, a, b, R_1 and R_2. The origin of coordinate system is set at the corner of the panel on the mid-plane. Parallel to the right-hand set of axes (X, Y, Z), in which X and Y are in the directions of the lines of curvature of the middle surface and Z is in the direction of the inward normal to the middle surface, the panel displacements are designated by $\overline{U}$, $\overline{V}$ and $\overline{W}$. The foundation is assumed to be a compliant foundation, which means that no part of the panel lifts off the foundation in the large deflection region. The panel–foundation interaction is represented by Pasternak model and the load–displacement relationship is assumed to be $p_0=\overline{K}_1\overline{W}-\overline{K}_2\nabla^2\overline{W}$, where p_0 is the force per unit area, $\overline{K}_1$ is the Winkler foundation stiffness and $\overline{K}_2$ is the shearing layer stiffness of the foundation, and $\nabla^2=\partial^2/\partial X^2+\partial^2/\partial Y^2$ is the Laplace operator. The panel is exposed to elevated temperature and is subjected to a transverse uniform pressure q. The CNTRC panel is made of a mixture of single-walled carbon nanotubes (SWCNTs) and the matrix which is assumed to be isotropic. The SWCNT reinforcement is aligned in the X direction and is either uniformly

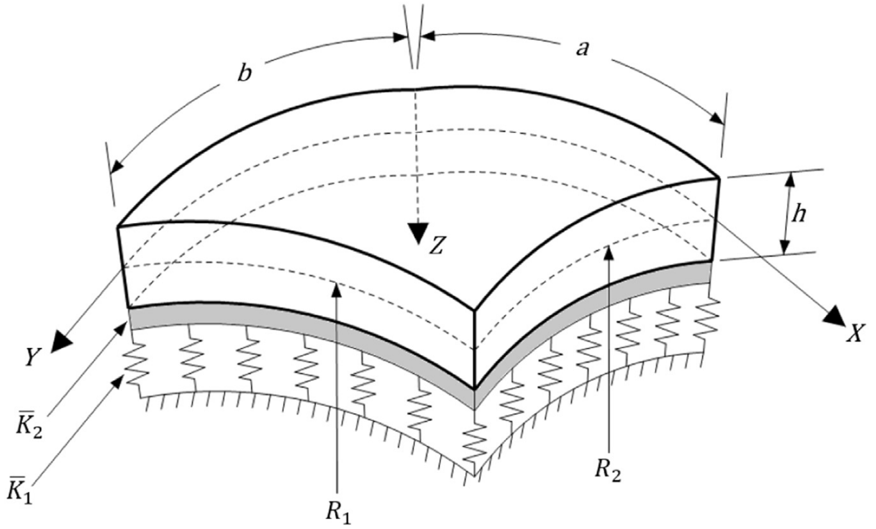

Fig. 1. Geometry and coordinate system of a double curved panel resting on a Pasternak elastic foundation.