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An extended finite element model for structural analysis of cracked beam-columns with arbitrary cross-section

M.R. Shirazizadeh [a,*], H. Shahverdi [b]

[a] Department of Mechanical and Aerospace Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran
[b] Department of Aerospace Engineering and Center of Excellence in Computational Aerospace, Amirkabir University of Technology, 424 Hafez Avenue, Tehran 15875-4413, Iran

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ABSTRACT

In this paper, an efficient finite element model for the structural analysis of cracked beam-columns with arbitrary cross-sections is presented. For this propose the combination of a planar beam element with only displacement degrees of freedom and extended finite element method (XFEM) is used. This element is highly capable in modeling the effect of edge cracks on the structural behavior of beams under various loading and boundary conditions. The method is able to solve the problem related to cracked beams with non-rectangular cross-sections with much less computational efforts compared to the available extended finite element models. Numerical examples are given to demonstrate the accuracy and the efficiency of the present method. There is a good agreement between the results obtained by the proposed method and those obtained by ABAQUS XFEM.

1. Introduction

Beam structures have been widely used in civil, mechanical and aerospace industries. The existence of damages like edge cracks can totally change the behavior of beam structures and reduces their safety. Therefore, developing reliable models to study the structural behavior of cracked slim structures has been considered by many researchers.

The local flexibility approach which is first introduced by Okamura et al. [1] is one of the most important ones of these models. In this method an edge crack is considered as a source of local flexibility. Thus the cracked beam is modeled as two intact parts connected each other with a massless rotational spring at the crack position. The stiffness of the rotational spring is expressed in terms of stress intensity factors utilizing fundamental theory of linear elastic fracture mechanics. This method has been implemented by many researchers to study various kinds of structural analysis [2–15]. The method provides an analytical solution to the described problem. However, it is only applicable for structures with simple geometries and boundary conditions. To resolve this problem, some researchers have focused on developing especial finite element models based on the concept of local flexibility for studying practical engineering problems [16–21].

In spite of the simplicity of the local flexibility method, there are two main deficiencies limiting its usage.

Firstly, as mentioned in the paper by Ebrahimi et al. [22], this method is only able to estimate the relationship between force and displacement in the region far from the crack. Therefore it is not suitable for stress analysis. And secondly, as noted earlier the stiffness of rotational spring is expressed in terms of stress intensity factor (SIF). Numerous numerical and empirical expressions for SIF's have been established for different forms of cracked bodies. For example, a method introduced by Li et al. [23] for determining stress intensity factors in cracked shaft could be mentioned. But closed-form expressions for SIF related to cracked beam structures have only been reported for limited kinds of cross-sections. Therefore the local flexibility method is not applicable for analyzing beams with arbitrary cross-section.

To overcome the mentioned problems, using a continuous displacement and stress field rather than the local flexibility model may be an option. Christides and Barr [24] introduced a continuous theory of cracked beam structures. They showed that the existence of cracks would significantly change linear distribution of stress and strain in the cracked section. They suggested two independent nonlinear functions to approximate distribution of stress and strain in the cracked region. However, their suggested functions are incompatible and the model can only be utilized for specific applications such as determination of the fundamental natural frequency of the beams.

Later Chondros et al. [25] and Shen and Pierre [26] introduced some improved approximations of stress and displacement field.

* Corresponding author. Tel.: +98 26 34474789.
E-mail addresses: mshirazizadeh@yahoo.com (M.R. Shirazizadeh), h_shahverdi@aut.ac.ir (H. Shahverdi).

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Their approximation established an accurate theoretical stiffness model of a cracked beam. However, the complexity of solutions makes the method unsuitable for practical applications. Also, to compute some of parameters in the crack disturbance functions the closed-form expressions for SIF are required. Therefore this method is only applicable for beams with limited kinds of cross-sections. To overcome this limitation, experimental or numerical simulations are required to determine stress distribution function. This makes the method expensive and time consuming. So the method cannot cover all weaknesses of the local flexibility approach.

Finite element method is known as one of the most powerful tools for modeling wide range of engineering problems. Standard 2 and 3D finite element method (FEM) can model discontinuities in structures through implementing very fine grids and singular elements around the crack tip. However, the task of constructing an acceptable mesh could be tedious if the cracked body has a complex geometry. In addition, a high computational cost of the method is a serious problem when more complicated problems such as crack growth are considered.

The XFEM is one of the most important refinements in the standard FEM to handle fracture mechanics problems with minimum computational task and remeshing process. This method presented by Belytschko and Black [27] based on the incorporation of special local enrichment functions into a standard finite element formulation. In recent decade this method is widely used to study various aspects of fracture mechanics [28]. A comprehensive review can be found in the published paper by Yazid et al. [29].

The main objective of this paper is to use the XFEM for structural analysis of cracked beams with arbitrary shape of cross-section. For this propose a Timoshenko beam element with only displacement degrees of freedom is coupled with partition of unity enrichment. The presented method can model the bending behavior of cracked beam with only one element through the thickness. Further, it is provided to model cracked beams with arbitrary cross-section without using 3D solid elements. Therefore, the computational efforts would be decreased compared to available standard and extended finite element models. Despite low computational costs, the method avoids limitations of the local flexibility and continuous cracked beam methods in analyzing beams with complex cross-sections. In addition, in the contrast to the local flexibility approach, the method is able to estimate nonlinear distribution of stress through the cracked beam section with good accuracy. Also, the presented model is suitable for the modal and buckling analysis of cracked beams. Numerical examples offered to validate the effectiveness and accuracy of present method.

2. Beam element with only displacement degrees of freedom

In this section a special class of beam elements which has only displacement degrees of freedom is explained. This element was introduced by Kwon and Bang [30] to model intact beams with rectangular cross sections. Here, the construction of XFE model based on this element is described for modeling a beam with edge crack. Fig. 1 shows the geometry and degrees of freedom of the element. As evident, the element has quadrilateral shape with two nodes on each side and each node has three degrees of freedom, two degrees of freedom along the x axis (u^b and u^t) and one degree of freedom along the y axis (v). Subscripts t and b represent the upper and lower edges of the element, respectively.

The element nodal displacement vector is defined as follows:

$$\{d\}_e = \begin{Bmatrix} u_1^b & u_1^t & v_1 & u_2^b & u_2^t & v_2 \end{Bmatrix}^T \tag{1}$$

And the element displacement field is expressed as

$$U = \begin{cases} u(x, y) \\ v(x) \end{cases} \tag{2}$$

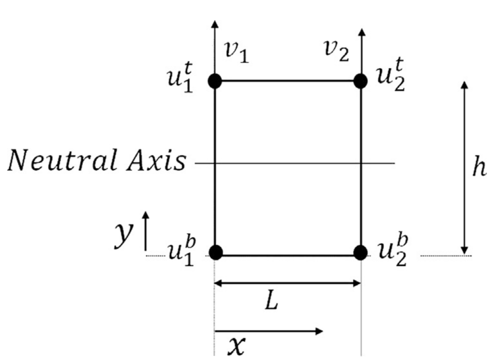

Fig. 1. Beam element with only displacement degrees of freedom.

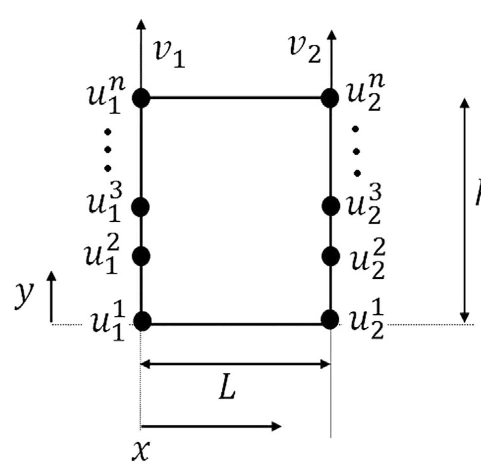

Fig. 3. Elements with additional degrees of freedom in the axial direction.

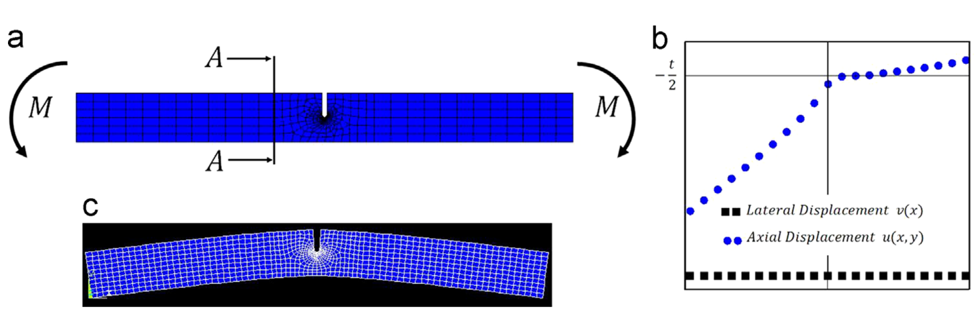

Fig. 2. Cracked beam under bending: (a) finite element model; (b) deformed shape and (c) displacement trough section A-A.