International Journal of Mechanical Sciences 99 (2015) 40–47

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Vibration of simply supported beams under a single moving load: A detailed study of cancellation phenomenon

C.P. Sudheesh Kumar, C. Sujatha*, K. Shankar

Machine Design Section, Department of Mechanical Engineering, Indian Institute of Technology Madras, Chennai 600036, India

ARTICLE INFO

Article history:
Received 1 December 2014
Received in revised form
7 April 2015
Accepted 2 May 2015
Available online 9 May 2015

Keywords:
Dynamic response
Resonance
Cancellation
Free vibration
Moving load
Railway bridge

ABSTRACT

Dynamic behaviour of simply supported uniform beams subjected to a single moving point load is analysed in this paper. A simple closed-form expression for free vibration response, valid for both lightly and heavily damped beams, is formulated. A detailed investigation of the cancellations of free responses is carried out. New interpretations for the cancellation mechanism, from the perspectives of free vibration amplitude and phase angle, are presented. Expressions for cancellation speed ratios are formulated based on the initial velocity and displacement conditions for free vibration. The effect of damping on the cancellations of free responses is also studied.

1. Introduction

Dynamic analysis of beams under moving loads has been an active area of research for the past several years. Structural integrity of bridges has been a cause of concern to engineers, especially, when traversed by high speed trains. With modern transportation becoming faster and heavier, this area of research continues to be an interesting and challenging one.

Numerous works on vibration of simply supported beams under moving loads are found reported in the literature. Fryba [1] conducted a comprehensive study on the vibrations of simply supported uniform beams due to the excitation of different types of moving loads. A discussion on the fundamental aspects of moving load problem was presented and a comparative study of the analytical and finite element solutions was given by Olsson [2]. The vertical acceleration response of a simple beam subjected to the passage of a single load was studied by Yau et al. [3] by assuming light damping ($\zeta < 0.03$). Law et al. [4] analysed the identification of moving force in both time and frequency domains. Lu et al. [5] reported that the dynamic response of a railway bridge to single load excitation is mainly influenced by the frequency characteristics, the so-called driving frequencies, of the train load.

Resonances and cancellation phenomena in beams under moving loads have been studied and reported by several researchers. Yang et al. [6] illustrated that a good design of railway bridges is the one that ensures suppression of the first resonance at all times by varying either the span length or the cross-section and showed that when the span to car length ratio equals 1.5 or 0.5 no resonant response will be induced on the beam. Savin [7] analysed the dynamic amplifications due to forced and free vibrations and showed that for some optimal span lengths corresponding to the wavelengths of the load, the responses can be cancelled theoretically. Yang et al. [8] studied analytically and showed that elastically supported beams have lower resonant (real) speeds compared to simply supported beams but their cancellation (real) speeds are close to those of beams with simple supports. They also showed that, at resonance speeds, damping does not affect the response in beams with elastic bearings. Yau et al. [9] illustrated that at cancellation condition, the residual responses caused by all previous loads which have traversed the beam will be suppressed and the response will be decided solely by the loads still acting on the beam.

Xia et al. [10] investigated the resonance mechanisms and conditions of train-bridge interaction and observed that the resonance mechanism is affected by the span, lateral and vertical stiffness of the bridge, axle arrangements and natural frequencies of the vehicles. The resonant response of a steel girder bridge traversed by high speed trains was investigated by Li et al. [11] with and without considering vehicle–bridge interaction. Ju et al. [12] analysed the resonant characteristics of multi-span bridges and proposed that a suitable axial stiffness between two simple beams will reduce the resonant vibrations at near resonance-conditions. Cho et al. [13] studied the resonance and cancellations mechanisms occurring in bridges subjected to high speed trains

* Corresponding author. Tel.: +91 44 22574682.
E-mail address: sujatha@iitm.ac.in (C. Sujatha).

http://dx.doi.org/10.1016/j.ijmecsci.2015.05.001
0020-7403/

and arrived at the following conclusions: resonance is not related to mode shape, whereas, cancellation depends on it. Euler–Bernoulli beam model with various boundary conditions was considered in the analysis and an optimal span length which suppresses resonance was proposed. Yau et al. [14] showed that resonant response can be induced in the bridge by a train within the operating speed range and if the characteristic length, instead of span length, is used for the continuous beam then the responses of both the simple and continuous beams will be at their peaks.

An illustrative interpretation of the cancellation phenomenon occuring in a beam under a single moving load, in terms of the homogeneous and particular solutions, was given by Museros et al. [15]. Pesterev et al. [16] predicted the speeds for which the amplitudes of free vibration response of a simply supported beam become minimal.

Xia et al. [17] showed that bridge damping has an influence on the cancellation effect; cancellation efficiency will be reduced with increase in damping. They reported two types of cancellation phenomena, the first being due to a single load and the second being related to the spatial intervals between the loads. Also, it was shown that free vibration of a bridge induced by a single load has inherent cancellations at some speeds and these are known as the first cancellation conditions. A novel method of suppression of resonance phenomenon of high speed railway bridges by inserting size-adjusted vehicles into the existing train arrangement was proposed by Shin et al. [18].

In most of the analyses, only lightly damped or undamped beams are considered. But, it has been reported that the conventional damping ratio of 5% used is too conservative for short span bridges [1,19]. So, the assumption of light damping is not valid in the case of bridges of short span.

In this paper, a simple and compact formula to determine the free vibration responses of a uniform beam, applicable to both lightly and heavily damped beams, is proposed for the first time. Also, a thorough investigation of the cancellation phenomenon is done and the effect of damping on the cancellation phenomenon is predicted. Though similar studies have been presented by many researchers in the past, a detailed description and interpretation of the cancellation mechanism has not been found reported.

2. Uniform beam with a single moving point load

2.1. Forced vibration

The general equation of motion of a simply supported beam of rectangular cross-section as shown in Fig. 1 traversed by a single force P at constant speed v is given by

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+c\frac{\partial w(x,t)}{\partial t}+\mu\frac{\partial^2 w(x,t)}{\partial t^2}=P\delta(x-vt) \tag{1}$$

where E is the Young's modulus, I is the moment of inertia of the cross-section, $w(x,t)$ is the transverse deflection of the beam, c is the damping coefficient, μ is the mass per unit length, δ is the Dirac delta function and x is the distance from one support in the direction of motion. L is the length of the beam and h_u is its height.

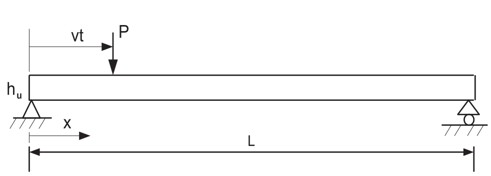

Fig. 1. Simply supported uniform beam with a load P moving with velocity v.

Assuming the solution in the form of mode superposition, the transverse deflection of the beam can be written as

$$w(x,t)=\sum_{n=1}^{\infty} q_n(t)\sin\frac{n\pi x}{L} \tag{2}$$

where $q_n(t)$ represents the generalized coordinate for nth vibration mode and $\sin(n\pi x/L)$, the corresponding mode shape.

Multiplying both sides of Eq. (1) by $\sin(n\pi x/L)$ and integrating from 0 to L, the generalised equation of motion of the beam is obtained as

$$\ddot{q}_n(t)+2\zeta_n\omega_n\dot{q}_n(t)+\omega_n^2 q_n(t)=\frac{2P}{\mu L}\sin\frac{n\pi vt}{L} \tag{3}$$

where ω_n^2 is the square of the natural frequency for nth mode given by

$$\omega_n^2=\frac{n^4\pi^4}{L^4}\frac{EI}{\mu} \tag{4}$$

and ζ_n is the corresponding damping factor. The normalised mid-span deflection for $t<L/v$, for zero initial conditions, is given by

$$\frac{w_{\text{forced}}(t)}{w_{\text{static}}}=\sum_{n=1}^{\infty}\frac{1}{n^4\sqrt{(1-K_n^2)^2+(2\zeta_n K_n)^2}}\left\{\sin(K_n\omega_n t).\right.$$
$$\left.-\frac{K_n}{\sqrt{1-\zeta_n^2}}e^{-\zeta_n\omega_n t}\sin\left(\omega_n\sqrt{(1-\zeta_n^2)}t\right)\right\} \tag{5}$$

where $K_n=\Omega_n/\omega_n$ is the non-dimensional speed and $\Omega_n=n\pi v/L$ is the excitation frequency. Here, $w_{\text{static}}=2P/(\mu L\omega_1^2)=2PL^3/(EI\pi^4)\approxeq PL^3/(48EI)$ is the static deflection of the mid-span of the beam where ω_1, hereafter referred to as ω, is the fundamental natural frequency of the beam.

2.2. Free vibration

Analysis of free vibration is important as it plays a major role in determining the modal parameters such as damping. It helps to identify the speeds with which the load (vehicle) should traverse the beam (bridge) so that there are no responses due to free vibration.

The free vibration response of the mid-span of the beam for $t>L/v$ is as follows:

$$q_n(t)=\left[q_{0n}\cos\omega_{dn}t+\left(\frac{\dot{q}_{0n}+\zeta_n\omega_n q_{0n}}{\omega_{dn}}\right)\sin\omega_{dn}t\right]e^{-\zeta_n\omega_n t} \tag{6}$$

where ω_{dn} is the damped natural frequency, q_{0n}, $\dot{q}_{0n}$ are the initial displacement and initial velocity of the mid-span respectively. These initial conditions are given by the displacement and velocity values of forced responses at time $t=T=L/v$ i.e., the moment the load departs the beam.

$$q_{0n}=-\frac{F_o e^{-\zeta_n n\pi/K_n}}{\omega_n^2\sqrt{(1-K_n^2)^2+(2\zeta_n K_n)^2}}\sin\left(\frac{n\pi}{K_n}\sqrt{1-\zeta_n^2}\right) \tag{7a}$$

where $F_o=2P/\mu L$.

$$\dot{q}_{0n}=\frac{F_o}{\omega_n^2\sqrt{(1-K_n^2)^2+(2\zeta_n K_n)^2}}\left\{K_n\omega_n\cos n\pi\right.$$
$$\left.-K_n\omega_n e^{-\zeta_n n\pi/K_n}\cos\left(\frac{n\pi}{K_n}\sqrt{1-\zeta_n^2}\right)\right\}$$
$$+\frac{\zeta_n K_n\omega_n}{\sqrt{1-\zeta_n^2}}e^{-\zeta_n n\pi/K_n}\sin\left(\frac{n\pi}{K_n}\sqrt{1-\zeta_n^2}\right) \tag{7b}$$

Substituting the values of q_{0n} and $\dot{q}_{0n}$ from Eqs. (7a) and (7b) in